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Subpixel edge detection for dimensional control by artificial vision

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Abstract. *Dimensional control by artificial vision is becoming a standard tool for industrialists interested in such remote and without contact measurement methods. The expected accuracy of those systems is dependent on camera resolution. High precision requires very costly charge coupling device sensors and frame grabbers. The proposed method tends to increase significantly the precision of dimensional measurements without increasing the hardware complexity. This algorithm is also quite robust against noisy images as it can be encountered in real world imaging; a precision of 1/16 pixel can easily be obtained with signal to noise ratio=2 dB. Our approach aims at improving the edge detection process involved in dimensional control by artificial vision. A lot of edge detection techniques with pixel resolution are well known and some of them are designed in order to be robust against image corruption. On the other hand B-spline interpolation methods have been considerably improved and popularized by the signal processing techniques proposed by M. Unser et al. An algorithm resulting from the merging of these two ideas is proposed in this paper. In this algorithm, the interpolation is prepared by an optimized filtering and by a detection of local maxima of gradient.*
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1 Introduction

Subpixel edge detection and localization in noisy images is a commonly needed tool for artificial vision applications; particularly for on-line dimensional control of manufactured parts. A lot of edge detection techniques with pixel resolution are well known and some of them are designed in order to be robust against image corruption. An interesting method has been initiated by Canny[1] and optimized linear filters using a similar approach are now quite classical in the image processing toolbox.[2–4] On the other hand, B-spline interpolation methods have been considerably improved and popularized by the signal processing techniques proposed by Unser *et al*.[5,6] An algorithm resulting from the merging of these two ideas is proposed in this paper. In this algorithm, interpolation is prepared by optimized filtering and by a detection of local maxima of gradient.

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It should be noted that this method is consistent with the continuous optimization approach of Canny, so that this edge detection algorithm should give optimal results with respect to Canny's criteria,[7] and a fair robustness against noise can be expected. In Sec. 2 of this paper, the method used for estimating a gradient with subpixel resolution is described. Section 3 is dedicated to a presentation of the edge detection and localization algorithm. Finally some experimental results are presented in Sec. 4.

2 Subpixel Gradient

2.1 *Optimized Filtering*

The approach proposed here is based upon an estimation of the spatial gradient. It is well known that derivation of noisy discrete signal is an ill-posed problem and that some regularization is needed. In the way initiated by Canny,[1] the idea is to base it on the optimization of some criteria; for instance, signal over noise ratio (SNR), edge localization, and nonmultiplicity of the responses. The result of the optimization process is a separable linear filter described by two impulse responses in the continuous domain, the implementation of which can be done in a recursive way, after sampling. In the same manner some other authors[2,3,8,9,4] have proposed first, second, or third order IIR filters, for performing an estimate of spatial gradient. The differences and results between these approaches are consistent with the chosen criteria. The method described here does not depend on this choice, and in this paper the simplest one is used; a first order operator proposed by Shen and Castan[4] is defined by

$$g(x)=ce^{-\alpha|x|}, \tag{1}$$

$$f(x)=\mathrm{sign}(x)de^{-\alpha|x|}, \tag{2}$$

$g(x)$ being the impulse response of the regularization filter and $f(x)$ being the impulse response of the derivative filter. After sampling and normalization, transfer functions are given as

$$F(z)=(e^{-\alpha}-1)\cdot(z^{-1}-z)\cdot\frac{1}{1-e^{-\alpha}z^{-1}}\cdot\frac{1}{1-e^{-\alpha}z}, \qquad (3)$$

$$G(z)=(1-e^{-\alpha})^2\cdot\frac{1}{1-e^{-\alpha}z^{-1}}\cdot\frac{1}{1-e^{-\alpha}z}. \qquad (4)$$

$f[k]$ and $g[k]$ will further denote the corresponding sampled impulse responses. The unique parameter is α, it can be seen as the ''width'' of the filters and taking a large value for it leads to smoothing operators which favor SNR against localization. Therefore the setting of α, as it is shown further, depends on initial SNR of the image to be processed.

The optimization process leading to the above proposed operator is done for continuous signal and the optimality does not stand after discretization. This point has been shown by Demigny and Kamlé.[7] The interpolation process proposed in the next section allows the method to stay in continuous domain, therefore it can be assumed that the optimality stands.

2.2 *Subpixel Gradient Estimation*

The subpixel gradient estimation is first introduced in the one-dimensional (1D) case. If $e[k]$ is the initial discrete signal, following Unser[5] its continuous interpolated version is given by

$$e(x)=\sum_{k=-\infty}^{+\infty}u[k]\beta^n(x-k), \qquad (5)$$

where $\beta^n(x)$ is a B-spline function of order n (see Appendix A), and where the interpolation is such that: $e[k]=e(x)|_{x=k}$. The impulse response $f(x)$ is decomposed on a p-order B-spline basis as

$$f(x)=\sum_{k=-\infty}^{+\infty}h[k]\beta^p(x-k).$$

For a continuous gradient estimation one has: $s(x)=e*f(x)$, so that, after summation, permutation and using the convolution property of B-spline functions

$$s(x)=\sum_j h[j]\sum_i u[i]\beta^{n+p+1}(x-j-i). \qquad (6)$$

If discrete B splines are introduced:[5] $b^n[k]=\beta^n(x)|_{x=k}$, we can write

$$e[i]=\sum_{k=-\infty}^{+\infty}u[k]\beta^n(i-k)=u*b^n[i] \qquad (7)$$

and

$$u[i]=e*b^n_{-1}[i], \qquad (8)$$

$b^n_{-1}[i]$ being defined by its z transform: $B^n_{-1}(z)=1/B^n(z)$ if $B^n(z)$ is the z transform of $b^n[k]$ (an example

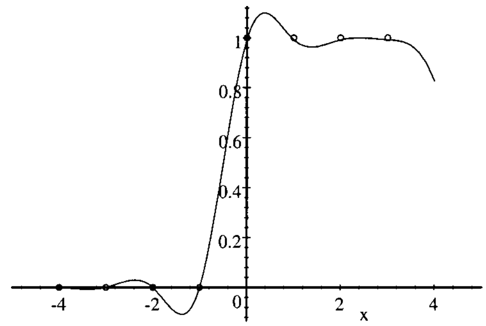


Fig. 1 Cubic B-spline interpolation of a discrete step.

of the cubic B-spline interpolation of a discrete step is presented in Fig. 1). The same expression can be obtained for $h[k]$ and finally, we can write

$$s(x)=\sum_k[(e*f*b^n_{-1}*b^p_{-1})[k]\beta^{n+p+1}(x-k)]. \qquad (9)$$

This operation can be decomposed in two steps, the first one consists in discrete convolution products

$$v[k]=(e*f*b^n_{-1}*b^p_{-1})[k] \qquad (10)$$

or in the z-transform domain

$$\frac{V(z)}{E(z)}=\frac{F(z)}{B^n(z)B^p(z)}. \qquad (11)$$

It has been demonstrated[5] that $B^n_{-1}(z)$ is always the transfer function of a stable linear filter (no pole on the unit circle). The second step of the operation described in Eq. (9), for a given x, is a summation on a finite number of integer values weighted by masks defined by B-spline functions of the order $n+p+1$

$$s(x)=\sum_k v[k]\beta^{n+p+1}(x-k). \qquad (12)$$

The interpolation must be done around the closest pixel; if the coordinate of this pixel is taken as axis origin, the x range is $[-1/2,+1/2]$. An n-order B-spline function has nonzero values only in the interval $\pm(n+1)/2$; therefore k must vary in the domain $\pm(n+p+2)/2$. This is illustrated in Table 1 where coefficient values are presented for an interpolation step of 1/16 pixel and $n+p+1=7$. The coefficients smaller than 10^{-4} are neglected.

The extension to a 2D signal (image) is easily performed in a separable manner

$$\nabla_x e(x,y)=\sum_{l=-\infty}^{+\infty}\sum_{c=-\infty}^{+\infty}v[l,c]\beta^{n+p+1}(x-c)\times\beta^{n+p+1}(y-l), \qquad (13)$$

where $v[l,c]$ is obtained by separable filtering of input signal following

Table 1 Interpolation masks ($n+p+1=7$).

step:x	$k=-3$	$k=-2$	$k=-1$	$k=0$	$k=1$	$k=2$	$k=3$
0	0.0002	0.0238	0.2363	0.4794	0.2363	0.0238	0.0002
0.0625	0.0003	0.0291	0.2578	0.4781	0.2153	0.0193	0.0001
0.1250	0.0005	0.0352	0.2795	0.4742	0.1950	0.0155	0.0001
0.1875	0.0007	0.0423	0.3014	0.4678	0.1755	0.0124	0.0000
0.2500	0.0009	0.0504	0.3230	0.4590	0.1569	0.0098	0.0000
0.3125	0.0013	0.0595	0.3442	0.4478	0.1395	0.0076	0.0000
0.3750	0.0018	0.0698	0.3647	0.4346	0.1231	0.0059	0.0000
0.4375	0.0025	0.0813	0.3843	0.4195	0.1080	0.0045	0.0000
0.5000	0.0034	0.0940	0.4026	0.4026	0.0940	0.0034	0.0000

$$\frac{V(z_1,z_2)}{E(z_1,z_2)}=\frac{F(z_2)G(z_1)}{B^n(z_1)B^p(z_1)B^n(z_2)B^p(z_2)}. \tag{14}$$

Interpolation masks $\beta^{n+p+1}(x-c)$ and $\beta^{n+p+1}(y-l)$ are also applied separably on rows then on columns.

All practical elements necessary to determine B splines and associated filters can be found in the papers of Unser *et al.*[5,6]

3 Edge Detection and Localization Algorithm

Our algorithm is a two stages process. A first approximation (at pixel resolution) of edge position is determined in the first step by detection of the local maximum of the spatial gradient estimated with an optimized operator (Shen-Castan[4] for instance). In the second step, a more precise determination of the maximum of the gradient is obtained by a local computation of the spline interpolation of order $n+p+1$ on the gradient image prefiltered by $1/B^nB^p$. This latter computation is limited to a width of half a pixel in the direction given by the local variation of the gradient. As shown in the previous section, the elementary operation in this step is an average weighted by a mask determined by $\beta^{n+p+1}(x)$, where x is the relative coordinate of the interpolation point in fraction of unity (see Table 1). For a rigorous 2D edge localization it is necessary to detect the gradient maximum along the direction of the gradient vector. Although it is possible to achieve such an algorithm, a less costly approach is proposed here. The position of the maximum of the greatest gradient vector component is taken as the edge position.

Proposition for an Algorithm

Detection

- Shen-Castan filter → Gradient estimation on the original pixel sampling grid
- Gradient local maxima detection → pixel-accuracy edge localization.

Subpixel localization

- Interpolation prefiltering $1/B^nB^p$
- Subpixel localization of max(max(row-gradient, column-gradient)).

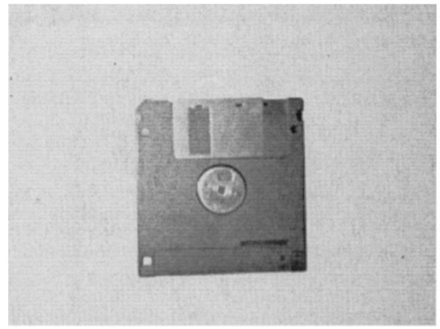


Fig. 2 Original microdisk image.

The proposed subpixel edge detection algorithm is less costly in terms of computation time compared to other standard methods (Haralick,[10] for instance) and it shows a great robustness against noise corruption. This last point and an estimation of the edge localization precision are illustrated in the next section. The global complexity of the algorithm (except for the stage of subpixel localization) is $O(n)$, n being the number of pixels of the image. The computation cost of the subpixel localization is approximately conversely proportional to the chosen interpolation step.

4 Experiments

The algorithm efficiency is tested on an image of a real object (a microdisk: image Figs. 2 and 3) having a straight side (right border of the microdisk) for which, physically, the best model is a straight line. The quality of the subpixel interpolation is estimated by a comparison between the edge point coordinates given by the algorithm and the best fitted (least mean square minimization) straight line equation. It should be noted that standard deviation of error takes into account edge delocalization as well as false edge points. Robustness against noise is estimated on the same image which is variously corrupted by an additive Gaussian white noise. The Shen-Castan operator (an example of a gradient image is presented in Fig. 4) has been chosen in this experiment. We have noticed during other tests that quite similar results are obtained with other optimum edge detectors (Canny-Deriche,[2] B–P–T).[11] The regularization parameter α has to be set according to the amount of noise added to the image.

Some experiments with various interpolation steps (see the curve in Fig. 5) have shown that for the chosen ex-

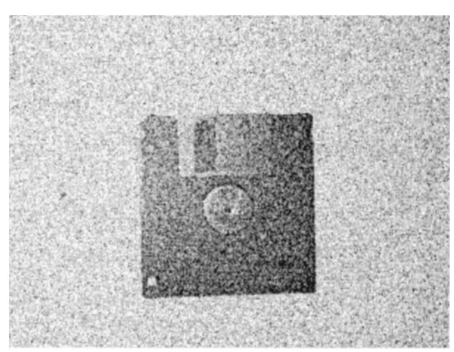


Fig. 3 Corrupted microdisk image: SNR=3 dB.

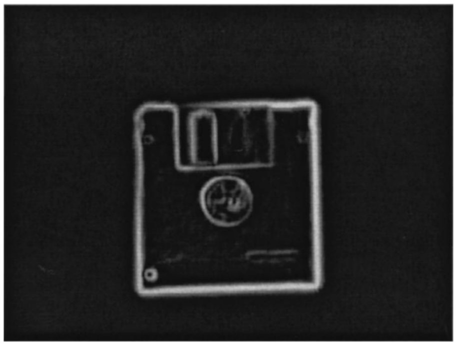


Fig. 4 Gradient image from Shen–Castan estimate.

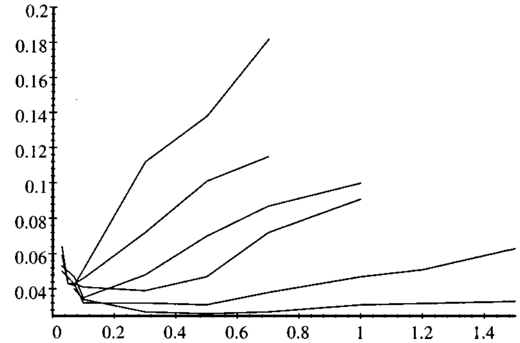


Fig. 6 Standard deviation vs α for various SNR; from the bottom to the top: ∞, 15, 8, 5, 2, 0 dB.

ample, a good tradeoff between complexity and accuracy is reached for a resolution of 1/16 pixel. For consistency, the orders of interpolation are the same for the filter and for the signal: $n=p=3$. In this case, the transfer function of the prefilter is given by

$$B_3(z)=\frac{z+4+z^{-1}}{6}. \tag{15}$$

Results are summarized on the curves of Fig. 6; these curves present the standard deviation σ between the detected edge points and the best straight line with respect to α and SNR. On the original image (SNR$=\infty$), σ is equal to 0.290 pixel for a one pixel resolution edge detection (without interpolation) where it is equal to 0.026 pixel for the subpixel algorithm (with an interpolation step of 0.0625 pixel). For noised images and for optimum setting (for α), the standard deviation remains smaller than the chosen interpolation step, even for highly corrupted images (SNR$=0$ dB). The efficiency of the algorithm is thus pointed out. The sensibility versus α is very low when SNR is high and the robustness is shown in Fig. 6. However, it should be noted that the tuning of the regularization parameter becomes more and more critical as the noise increases. As previously denoted, this parameter, which is inversely proportional to impulse response ''width,'' has to decrease as the noise increases. Figure 7 shows an example of the distribution of edge points detected at one pixel resolution and at subpixel (1/16 pixel) resolution. The approximation in the 2D approach leads to a slight discrepancy for 45° oriented edges; the curve of Fig. 8 illustrates this last point. Let us note finally that the experiment was led under standard MATLAB on a 350 MHz Intel Pentium II processor, the run of the prefiltering stage was taken as 0.61 s for a 256 $\times$256 image, whereas subpixel localization for the interpolation step of 1/16 pixel lasts 2.7 ms by edge point. According to the remark of the previous section, this time is divided by two for a step of 1/8 pixel.

5 Conclusion

An algorithm for edge detection and localization with subpixel resolution has been proposed. It is based on a B-spline interpolation of the initial signal and of an optimized derivative filter. The proposed method presents a guarantee of optimality according to Canny's or Shen's criteria and it leads to high immunity against noise, this point being particularly important for real world image processing and industrial applications. The operations involved consist mainly in phase linear recursive filtering of low order (1 or 2) and parameters, in very limited number (2), are not critical when noise is low. The efficiency of the algorithm is illustrated on an image of a real object featuring a straight physical edge so that a good detection of a

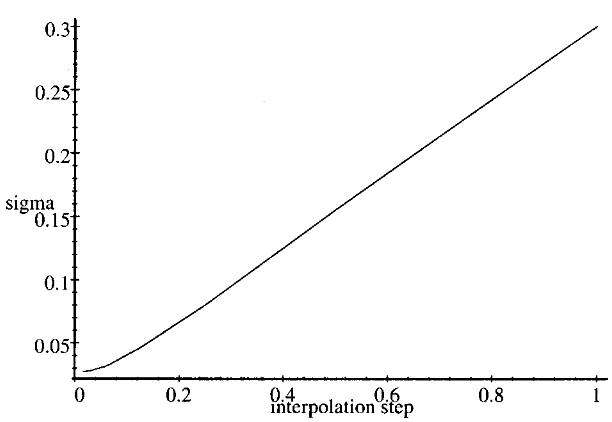


Fig. 5 Standard deviation vs interpolation step.

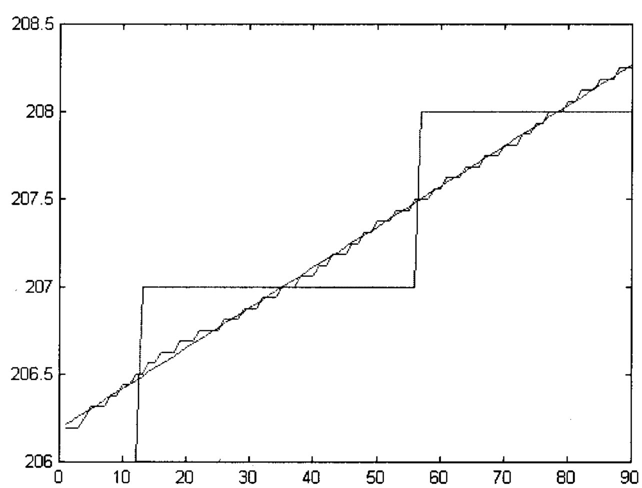


Fig. 7 Graph of the right border of the microdisk (pixel, subpixel and best straight line) for SNR$=\infty$.

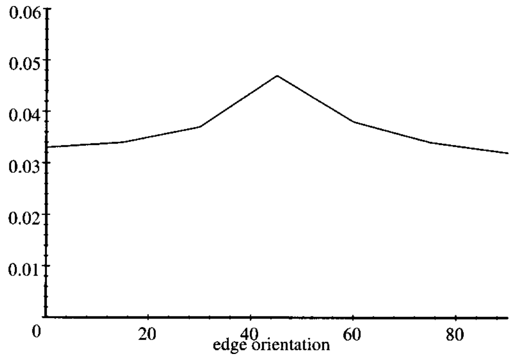


Fig. 8 Standard deviation vs edge orientation for SNR=15 dB.

straight line is a pertinent clue of quality. The Shen–Castan operator used in this example is effective for steep edge detection. In the case of a more blurred edge, Canny–Deriche or the B–P–T operator should give better results. A more rigorous approach of the 2D extension is under consideration.

An estimate of comparative performance of this algorithm and other subpixel edge detection methods is still to be done, but some criteria, which are pertinent for this sort of operator, must be first defined. An extensive study of subpixel measurements on well characterized real objects shot with a well calibrated imaging system is in progress and the preliminary results seem in good agreement with the performance presented in this paper.

Finally, it is interesting to remark that the application of this method with the goal of improving performance of edge detector designed in the continuous domain for pixel resolution would not lead to great results. The reason is that when the order of B-spline interpolation tends to infinity, the transfer function of the complementary filter tends to unity. This last point is demonstrated in Appendix B.

Appendix A: B-Spline Interpolation Filters

Following Unser,[5] the expression of B-spline function $\beta^n(x)$ is

$$\beta^n(x)=\sum_{j=0}^{n+1}\frac{(-1)^j}{n!}\binom{n+1}{j}\left(x+\frac{n+1}{2}-j\right)^n \cdot u\left(x+\frac{n+1}{2}-j\right), \qquad \text{(A1)}$$

where $u(x)$ is the Heaviside function and

$$\binom{n}{p}=C_p^n=\frac{n!}{p!(n-p)!}.$$

These functions can also be defined by a convolution product:

$$\beta^n(x)=\underbrace{\beta^0*\beta^0*\ldots*\beta^0(x)}_{n\ \text{times}},$$

where $\beta^0(x)$ is the box function.

Table 2 Transfer function of B-spline interpolation filters.

n	$B^n(z)$
0	1
1	1
2	$\frac{z+6+z^{-1}}{8}$
3	$\frac{z+4+z^{-1}}{6}$
4	$\frac{z^2+76z+230+76z^{-1}+z^{-2}}{384}$
5	$\frac{z^2+26z+66+26z^{-1}+z^{-2}}{120}$

The transfer function of the digital filter used for the interpolation is

$$B^n(z)=\sum_{k=-\infty}^{+\infty}b^n[k]z^{-k}, \quad \text{with} \quad b^n[k]=\beta^n(k), k\in\mathbb{Z}.$$

Transfer functions of B^n filters up to order 5 are given in Table 2.

Appendix B: Limit of B-Spline Interpolation

The interest of B-spline interpolation is mainly due to the ideal band limited signal limit attained when B-spline order tends toward infinity. This property is linked to the Gaussian limit of these functions. Indeed, Unser *et al.* have shown[12] that limit of $\beta^n(x)$ if n tends toward infinity is a Gaussian

$$\beta^n(x) \xrightarrow[n\to\infty]{} \frac{1}{\sqrt{2\pi\sigma_n^2}}\exp-\frac{x^2}{2\sigma_n^2}$$

with

$$\sigma_n^2=\frac{n+1}{12}. \qquad \text{(B1)}$$

If this limit is taken to estimate limit of $b^n*b^p(k)$, it follows:

$$b^n*b^p(k)\simeq\frac{1}{2\pi\sigma_n\sigma_p}\sum_i \exp-\left[\frac{i^2}{2\sigma_n^2}+\frac{(k-i)^2}{2\sigma_p^2}\right]$$

and with

$$l=i-k\frac{\sigma_n^2}{\sigma_n^2+\sigma_p^2}$$

and

$$\sigma^2 = \frac{\sigma_n^2 \sigma_p^2}{\sigma_n^2 + \sigma_p^2}$$

$$b^n * b^p(k) \simeq \frac{1}{2\pi\sigma_n\sigma_p} \exp - \frac{k^2}{2(\sigma_n^2+\sigma_p^2)} \sum_l \exp - \frac{l^2}{2\sigma^2}. \tag{B2}$$

If $n,p \to \infty$ therefore, $\sigma^2 \to np/12$ and $(1/\sigma)\Sigma_l \exp - (l^2/2\sigma^2)$ can be considered as a Riemann summation of step $1/\sigma$, and when σ tends to zero this summation converges to: $\int_{-\infty}^{+\infty} e^{-(x^2/2)} dx = \sqrt{2\pi}$; reporting in Eq. (B2), it is

$$b^n * b^p(k) \simeq \frac{1}{\sqrt{2\pi(\sigma_n^2+\sigma_p^2)}} \exp - \frac{k^2}{2(\sigma_n^2+\sigma_p^2)}.$$

In another point and with the same approximation, the limit of $\beta^{n+p+1}(x)$ with $x = k+\alpha$, α being the decimal part of x, is given by:

$$\beta^{n+p+1}(x) \simeq \frac{1}{\sqrt{2\pi(\sigma_n^2+\sigma_p^2)}} \exp - \frac{k^2}{2(\sigma_n^2+\sigma_p^2)} \times \exp - \frac{2k\alpha}{2(\sigma_n^2+\sigma_p^2)} \exp - \frac{\alpha^2}{2(\sigma_n^2+\sigma_p^2)}$$

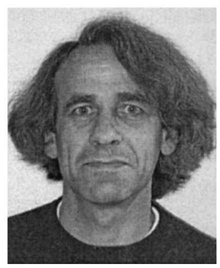
therefore when $n,p \to \infty$

$$\beta^{n+p+1}(k+\alpha) \simeq b^n * b^p(k) \exp - \frac{k\alpha}{\sigma_n^2+\sigma_p^2}. \tag{B3}$$

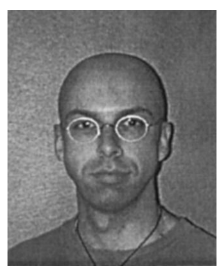
If the decimal part is null (pixel resolution), $\alpha = 0$: $b^{n+p+1}(k) \simeq b^n * b^p(k)$, and the effect of B-spline interpolation is null.

References

1. J. F. Canny, ''A computational approach to edge detection,'' *IEEE Trans. Pattern Anal. Mach. Intell.* **8**, 679–698 (1986).
2. R. Deriche, ''Using Canny's criteria to derive a recursively implemented optimal edge detector,'' *Int. J. Comput. Vis.* **1**(2), 167–187 (1987).
3. O. Laligant and F. Truchetet, ''Generalization of Shen-Castan and Canny-Deriche filters,'' *Robotics and Machine Perception, SPIE's Int. Technical Working Group Newsletter* **7**(2), 5–12 (1998).
4. J. Shen and S. Castan, ''An optimal linear operator for step edge detection,'' *Comput. Vis. Graph. Image Process.* **54**(2), 112–133 (1992).
5. M. Unser, A. Aldroubi, and M. Eden, ''B-spline signal processing: Part I—Theory,'' *IEEE Trans. Signal Process.* **41**(2), 821–833 (1993).
6. M. Unser, M. Aldroubi, and M. Eden, ''B-spline signal processing: Part II-Efficient design and applications,'' *IEEE Trans. Signal Process.* **41**(2), 834–848 (1993).
7. D. Demigny and T. Kamlé, ''A discrete expression of Canny's criteria for step edge detector performances evaluation,'' *IEEE Trans. Pattern Anal. Mach. Intell.* **19**(11), 1199–1211 (1997).
8. V. Stoclin and L. Duvieubourg, ''Discrete criteria for edge detection,'' in *Proc. Int. Symposium on Image Processing and Artificial Vision Systems, TISVA'98*, pp. 189–196, Oujda, Japan (1998).
9. A. Chehikian, ''Filtres récursifs pour l'estimation du gradient et la détection de contours par interpolation spline,'' *Traitement du Signal et des Images* **14**(1), 29–42 (1997).
10. R. M. Haralick, ''Digital step edges from zero crossing of second directional derivatives,'' *IEEE Trans. Pattern Anal. Mach. Intell.* **6**(1), 58–68 (1984).
11. E. Bourennane, M. Paindavoine, and F. Truchetet, ''Amélioration du filtre de Canny-Deriche pour la détection des contours sous forme de rampe,'' *Traitement du Signal et des Images* **10**(4), 297–310 (1993).
12. M. Unser, A. Aldroubi, and M. Eden, ''On the asymptotic convergence of B-spline wavelets to gabor functions,'' *IEEE Trans. Inf. Theory* **38**(2), 864–872 (1992).



F. Truchetet: biography appears with the paper "Machine vision systems in the metallurgy industry," in this issue.



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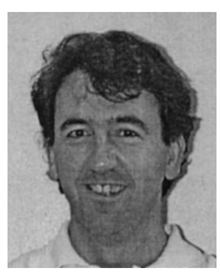

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