Visual Servo Control Using Orthonormal Polynomial

Hamid. TAIRI, Tadeusz. SLIWA, Yvon. VOISIN, 
Alain. DIOU and Larbi. RADOUANE *
Université de Bourgogne LE2I, IUT Le Creusot
12 rue de la Fonderie F-71200 Le Creusot France
Tel: +33 38 5 73 10 90
Fax: +33 3 85 73 10 97
e-mail: h.tairi@iutlecreusot.u-bourgogne.fr
* LESSI Faculté des sciences Dhar El Mahraz Fès Morocco

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Abstract:-This paper describes an application of the visual servoing approach to vision-based control in robotics. The basic idea addresses the use of a vision sensor in the feedback loop within the controlled vision framework. It consists in tracking of arbitrary 3-D objects travelling at unknown velocities in a 2-D space (depth is given as known). Once the necessary modeling stage is performed, the framework becomes one of automatic control, and naturally stability, performance and robustness questions arise. Here, we consider to track line segments corresponding to the edges extracted from the image being analyzed. Two representations for a line segment are presented and discussed, and an appropriate representation is derived. A SISO (Single Input Single Output) model for each parameter of a line segment is then derived and represented by an orthonormal Laguerre network put in state space form. The appeal of this new approach is that it eliminates the need for assumption about the plant order, the time delay and the unmodeled dynamics. For modeling by Laguerre filters, the system must be stabilizable. This problem is handled by an input output data filtering. Hence the poles of filtered model are relocated inside the unit circle. A simple adaptive predictive control is then used for its simplicity. To illustrate the advantages of using the Laguerre network associated to an adaptive input output data filtering, over the conventional control techniques, we carry out a comparison on simulated examples to a PID controller.

1 Introduction
In the domain of robotics, recent advances in vision sensor technology and vision processing have led to use vision data in the control loop of a robot [1-2]. It should be emphasized that this approach differs from the ones referred to as dynamic vision approaches, which exploit, without controlling it, the motion of the sensor or of the objects in the scene. In the visual servoing approach, the first step consists in defining in the image a particular set of characteristic features that constitutes the goal to be reached. The second step will be to design a control that will ensure convergence toward the configuration corresponding to the goal image by starting from a different initial condition, setting out two kinds of problems:
• The choice and the extraction of the visual feature elements to be used in the control
• The analysis and the synthesis of the control scheme from the point of view of automatic control theory.

For the first problem, it should be noted that the characteristic parameters allowing definition of the goal image were often selected in relation to the existence of algorithms allowing their extraction within a reasonably short time interval. This is why it is not surprising that most of the work performed used low-level primitives like contour points, segments [1, 3]. For the second problem, the most relevant results in the literature are due to [4-5]. Two kinds of studies have been conducted: the analysis of the mapping between the screw space of the camera motion and the space of velocity fields in the image, and the design of control schemes with the study of their stability and performance. Concerning the control schemes, it was shown that the performance of the used controllers including the PID and the LQG controllers were sensitive to errors in visual measurements, to additive noise and unmodeled dynamics and exhibit large oscillations in the system input and output, particularly during abrupt changes [1, 6]. Thus, the control performance degenerates. In fact this was expected since the
control synthesis is based on a prescribed model obtained under a set of assumptions which may be violated. The solution is obviously the search for a really robust control against unmodeled dynamics and noise. This problem is our main interest in the present work.

In recent years, there has been renewed interest in the use of orthogonal functions for system approximation, modeling, filtering, identification and control. Particularly, the discrete Laguerre functions have been considered, [7, 8-9]. One of the most important parameters in the Laguerre model is its pole location, or time scaling factor. If this parameter is selected suitably, then the Laguerre model can efficiently approximate a large class of linear systems [8, 10]. The major advantage of such an approach is that any stable system can be modeled without structural knowledge on the system order and phase.

In this paper, the problem of visual tracking is considered using line segments as features. Two representations of a line segment are presented and discussed, and an appropriate representation based on the parameters of line segment: midpoint, length, orientation and distance to origin is adopted. For each line parameter, we derive a SISO model. As this model is obtained under some simplification assumptions, we define a new good data model obtained by an input output data filtering. The proposed filter has the main design features: the signals are filtered by a low pass filter which is used to reduce the "high frequency" modes of the unmodeled dynamics, and a linear operator which allows to remove the system stability assumption. The filtered plant model is then described by a Laguerre network put in state space form, then state space control design techniques may be used. Here a predictive control scheme is used for its simplicity and robustness. In the second case, for a comparison purpose, we propose to use a PID controller with data filtering. The simulation results show the superiority of the proposed approach based on Laguerre series representation over the conventional control techniques.

The organization of the paper is as follows. A parametric representation for a 2D segment is described in Section 2. Modeling of the visual tracking is described in Section 3. Visual predictive Control using Laguerre network is detailed in Section 4. In Section 5, Mathematical model of the robot’s dynamics. Section 6 deals with the simulation results and Section 7 concludes the paper.

2 Parametric Representation for a 2D Line Segment

The classic representation is the Cartesian coordinates of the two end points. It is clear, that tracking both endpoints of each segment will be very difficult, since they are not at all reliable due to the fact that segments can be broken from one frame to another. For this reason, two types of representation have been considered.

- Representation $[c, D, \theta, \epsilon]^T$: In this representation, a segment having endpoints at points $P_1(x_1, y_1)$ and $P_2(x_2, y_2)$ is characterized by $[c, D, \theta, \epsilon]^T$. The components of this vector (distance of origin to the line segment, distance along the line from the perpendicular intersection to the midpoint of the segment, orientation and length) are derived from those of the endpoints.

- Representation $[x_m, y_m, \theta, \epsilon]^T$ where the point $P_m(x_m, y_m)$ defines the coordinates of the midpoint of the segment.

If we consider the effect of image noise on the process of extracting edge, we can make a number of observations:

1- The representation $[c, D, \theta, \epsilon]^T$ leads to a covariance matrix that depends strongly on the position of the associated line segment into the image. Therefore, two given line segments with the same length and orientation will have their uncertainty on the $(c, D)$ parameters completely different depending on their position within the image. This is a real drawback since a large value for $c$ (resp. $D$) will lead to a large uncertainty for $D$ (resp. $c$). This is not the case for the midpoint representation since the uncertainty associated to the midpoint depends only on the uncertainty on the endpoints.

2- The perpendicular position of a line segment may be measured with precision. This value is often a parameter of the segment extraction process, and is usually of the order of a pixel.

3- The orientation of a line segment has a precision proportional to the ratio of the perpendicular precision to the length. Longer segments have a more precise orientation. Then, from these
representations, we ultimately settled on a redundant representation composed of five independently estimated parameters: \([x_m, y_m, \theta, \ell, c]^T\). The extra parameter \(c\) provides the perpendicular tolerance for matching, as well as a direct estimation of perpendicular velocity of the segment.

3 Modeling of the Visual Tracking Problem

We will develop the mathematical model for the 2-D visual tracking of an object. The 2-D visual tracking of an object is realized by visually tracking feature segments that belong to the object. Each segment is characterized by \((x_m, y_m, \theta, \ell, c)\).

Under two assumptions [2, 3]:

• The disturbances are deterministic and constant
• The noise terms are neglected

The mathematical model [6] (ARMA) described by:

\[ A(q^{-1})Y(k) = q^{-d}B(q^{-1})U(k) + \xi(k) \]  

with \( A(q^{-1}) = (I + a_1q^{-1} + a_2q^{-2})I_5 \), \( B(q^{-1}) = (b_0 + b_1q^{-1})I_5 \), \( q^{-d} \) is the backward shift operator, \( d \) is the time delay, \((d = 1)\), \( Y(k) = (y_1(k), y_2(k), y_3(k), y_4(k), y_5(k))^T = (x_m(k), y_m(k), \theta(k), \ell(k), c(k))^T \) is the output vector,

\( U(k) = (u_1(k), u_2(k), u_3(k), u_4(k), u_5(k))^T = (u_x(k), u_y(k), u_q(k), u_r(k), u_c(k))^T \) is the control input vector,

\( \xi(k) = (\xi_1(k), \xi_2(k), \xi_3(k), \xi_4(k), \xi_5(k))^T \) is a white noise vector.

In the following, we will develop a robust visual tracking scheme using the Laguerre network representation in the state space form. The multivariable model (1) may be decomposed into five decoupled SISO systems as in [1, 3]. So we can work with five SISO models.

For the synthesis of our predictive control, we assume any SISO system of (1) is under the following assumptions:

• **A1** There exists a scalar \( \rho \in \mathbb{R}^+ \) such that: \( \| \Theta_i \| \leq \rho \) where \( \Theta_i = \begin{bmatrix} -a_1, -a_2, -b_0, b_1 \end{bmatrix}_{i=1, \ldots, 5} \).

• **A2** There exists a non-negative scalar \( \mu \) such that: \( \| \xi_1(k) \| \leq \mu z_i(k) \) with \( i = 1, \ldots, 5 \) where \( z_i(k) \) is a normalizing signal that may be defined by: \( z_i(k) = \sigma z_i(k-1) + \max_{0 \leq j \leq 1} \| \phi_i(k-j) \| \), \( \phi_i^T = [y_1(k-1), y_1(k-2), u_1(k-1), u_1(k-2)] \), \( z_i(0) > 0, 0 < \sigma < 1 \) and \( z_{i0} \geq 0 \) are arbitrary chosen.

• **Input output data filtering.**

A key issue to develop a robust control is the definition of a good data model associated to a robust parameter estimation algorithm. The "good data" model is defined as follows. Let the \( \Pi \) linear operator be defined by:

\[ \Pi(\cdot)(k-t) = \delta^{t+i}(.)(k-t), \quad 0 < \delta < 1 \]  

Given two asymptotically stable \( O(q^{-1}) \) and \( P(q^{-1}) \) then operating on system (1) by the filter \( \Pi(O(q^{-1})/P(q^{-1})) \) leads to:

\[ A_\delta(q^{-1})Y^\prime(k) = q^{-d}B_\delta(q^{-1})U^\prime(k) + \xi^\prime(k) \]  

where, \( O(q^{-1})Y(k) = P(q^{-1})Y^\prime(k) \), \( O(q^{-1})U(k) = P(q^{-1})U^\prime(k) \) and \( O(q^{-1})\xi(k) = P(q^{-1})\xi^\prime(k) \)

\[ Y^\prime(k) = \delta Y^\prime(k), \quad U^\prime(k) = \delta U^\prime(k), \quad \xi^\prime(k) = \delta \xi^\prime(k), \quad A_\delta(q^{-1}) = (1 + a_1 \delta q^{-1} + a_2 \delta^2 q^{-2})I_5, \quad B_\delta(q^{-1}) = (b_0 \delta + b_1 \delta^2 q^{-1})I_5 \]
**Lemma 1.** Consider the filtered model (3), subject to assumption \( A_1 \), then for \( |\delta| \leq \frac{1}{\rho \sqrt{2}} \), \( A\delta(q^{-1}) \) is a matrix whose diagonal elements are stable polynomials.

**Lemma 2.** Consider the filtered model (3) subject to \( A_2 \), then there exists a scalar \( \mu > 0 \) such that

\[
\left\{ \begin{array}{l}
\xi(k) \leq \mu z_i(k) \\
\mu < \mu
\end{array} \right.
\]

with \( z_i(k) = \sigma z_i(k-1) + \max \left\{ \Phi_i(k-1) z_{i0} \right\} \)

\[
\Phi_{i}^T = [y_i(k-1), y_i(k-2), u_i(k-1), u_i(k-2)] \quad 0 < \sigma < 1, \ z_{i0} > 0 \ \text{and} \ z_i(0) > 0 \ \text{are arbitrary chosen.}
\]

**Proofs:** (final manuscript)

Thus the filter \( \Pi(O/P) \) permits also to reduce the effects of the unmodeled dynamics.

### 4 Predictive Control Using Laguerre Network

One way to design a robust adaptive controller requiring minimal a priori information is to abandon the system transfer function models and instead develop an unstructured adaptive control scheme using an orthonormal series. The set of Laguerre functions is particularly appealing because it is simple to represent, is similar to transient signals and exhibit strong features in identifying time delay because of its similarity to pad approximates [8-9].

In continuous time, the Laguerre functions are defined as [8]:

\[
l_j(t) = \sqrt{2p} \exp(pt) dt_j^{-1} \{ t^j \exp(-2pt) \}
\]

where \( j \) is the order of the function \( j = 1, ..., N \) and \( p \) is the time scale factor.

A convenient representation of Laguerre filters in the frequency domain is defined as:

\[
l_j(s) = \sqrt{2p} \frac{(s-p)^j}{(s+p)^j}, \quad j = 1, ..., N \], where \( l_j(s) \) is the Laplace transform of \( l_j(t) \) [8].

The orthonormality is preserved in the s-domain and this is generated by the simple and convenient Ladder network of Fig 1.

The Laguerre representation in state space form for each parameter of a line segment is:

\[
L_i(k+1) = A_i L_i(k) + b u_i(k), \quad i = 1, ..., 5
\]

where the states of the Laguerre Ladder Network are defined as: \( L_i(k) = [l_1(k), ..., l_N(k)] \)

\( A_i \) is an \((N \times N)\) matrix and \( b \) is an \((N \times 1)\) vector.

\[
A_i = \begin{bmatrix}
\tau_1 & 0 & 0 \\
\mathrel{\vdots} & \ddots & \vdots \\
\tau_1 & \tau_2 & \tau_3 \\
\end{bmatrix}
\]

T is the sampling period

\[
\tau_1 = \exp(-pT), \quad \tau_2 = \frac{2}{p} \exp(-pT) - 1, \quad \tau_3 = -T \exp(-pT) - 2(\exp(-pT) - 1) \quad \text{and} \quad \tau_4 = \sqrt{2p} \left( 1 - \tau_1 \right) / p
\]
The above state space model is stable (p>0) and controllable. The filtered output of the process is then approximated by the weighted sum of the Laguerre filters.

\[ \hat{y}_i(k) = \hat{C}_i^T(k)L_i(k) \quad i=1,...,5 \]  

(5)

where \( \hat{C}_i(k) \) contains the Laguerre gains needed to represent the signal \( \hat{y}_i(k) \). The RLS is used to identify the parameter \( \hat{C}_i(k) \).

- Predictive control law for each SISO system

Our objective is to design an adaptive control law that will make the system input \( u_{ir}(k) \) and output \( y_{ir}(k) \) bounded for all the time. The previously derived state space, the n-steps ahead prediction for reference trajectory \( y_{ir}(k+n) \) are used. We finally get:

\[ u'_i(k) = y_{ir}(k+n) - \hat{y}_i(k) - \hat{\alpha}_i^T(k)L_i(k)\hat{\beta}_i(k) \]  

(6)

where \( \hat{\alpha}_i^T(k) = \hat{C}_i^T(k)(A^n-I) \), \( \hat{\beta}_i(k) = \hat{C}_i^T(k)(A^{n-1}+...+I)b \)

5 Mathematical Model of the Robot’s Dynamics  (final manuscript)

6 Simulation Results (final manuscript)

7 Conclusion

In this paper, we have considered the visual tracking problem. Specifically, we have been interested in tracking line segments corresponding to the edges extracted from the image being analyzed. We first proposed an adequate representation for a line segment and derived a MIMO model. This model may be decomposed into decoupled SISO models. An input output data filtering is then applied to each models. The filtered model is then represented by an orthonormal Laguerre network put in state space form and a simple predictive control law is synthesized. The appeal of this approach is that it eliminates the need for assumptions about the system plant order and permits to handle efficiently the unmodeled dynamics and noise. To show the effectiveness of the proposed approach, a comparison on simulated examples to a PID controller has been carried out.

References

Brief biography of principal author:


Hamid. TAIRO received his PhD degree in Image processing, Automatic and Computer Sciences from Sidi Mohamed Ben abbellah University, FES, MOROCCO, in March 2001. Currently post-doctoral in the Laboratory LE2I (Laboratoire d’Electronique, Informatique and Image) of the university of the Burgundy, IUT, Le Creusot France. His research interests include robotics, control, motion estimation and 3-D reconstruction.