Complex networks: application for texture classification

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ABSTRACT

This article describes a new method and approach of texture characterization. Using complex network representation of an image, classical and derived (hierarchical) measurements, we present how to have good performance in texture classification. Image is represented by a complex networks: one pixel as a node. Node degree and clustering coefficient, using with traditional and extended hierarchical measurements, are used to characterize "organisation" of textures.

Keywords: Image processing, texture analysis, complex networks

1. INTRODUCTION

Texture analysis have important role in numerous application of image processing. Many different approaches to texture analysis have been proposed. Among the most widely used texture measures are those derived from gray level cooccurrence matrices or difference histograms, "texture energy" measures obtained by local linear transforms, and features based on multi-channel Gabor filtering or Markov random field model.\textsuperscript{1,2}

Introduced recently,\textsuperscript{3–5} complex networks can be adapted to represent the relation and characterization between elements and become appropriate to characterize picture pattern. It is possible to represent an image as a complex network.\textsuperscript{6}

This paper overviews our approach, presents in the first part complex networks and image representation, in the second part methods that were used for comparison. The third part exposes complex networks method's results, with the efficiency of hierarchical measurements. The last part concludes with an overview of the obtained results and suggest possibilities for further improvements and complementary work.

2. IMAGE REPRESENTATION AND COMPLEX NETWORK

2.1. Complex network and measurements

A complex network is a set of nodes connected between them. One connection (or edge) between two nodes (or vertices) indicates an interaction between these two edges. Edges can be binary (i.e. presence or absence of connection) or weighted, and directed or not. The present work is limited to non-directed edges. All complex networks can be represented mathematically by a matrix called the adjacency matrix. With a complex network with $N$ nodes, the adjacency matrix ($W$) have a dimension $N \times N$. The weight of the connection from each node $j$ to each node $i$ $(i, j = 1, 2, ..., N)$ is represented as $W(i, j)$, with null value being assigned in the absence of such a connection. A second matrix $W_t$, binary, is also obtained. It contains only the most significant connections. For example, connections which are greater weight (only elements of $W$ which are greater than or equal to a threshold $T$ are kept); it can be seen in the example in Figure 1. The characterization of the topological and connectivity properties of complex networks can be achieved by using measurements borrowed from graph theory\textsuperscript{7} and complex network research\textsuperscript{5} including but being by no means limited to:

Degree and Strength: The degree of a given node is equal to the number of connections which it makes. For weighted connections the degree of a node is called strength and corresponds to the sum of all the weights of
Figure 1. (a): Weighted small complex network (b): the adgency matrix $W$, (c): $W_t$ matrix (binary) obtain with $T = 2$.

Figure 2. Illustration of degree (D) and clustering coefficient (C) calculation of the node represented in black: $D = 4$ and $C = \frac{3}{3 \times 3 \times 2} = \frac{1}{2}$.

the respective links. An example in Figure 2 illustrates this definition. The frequency histograms of the degrees (or strengths), as well as the respectively inferred average values, provide an important characterization of the connectivity of the network under analysis.

**Clustering Coefficient**: The clustering coefficient of a given node $i$ is defined as:

$$ C_i = \frac{\text{Number of connections between nodes connected to node } i}{\text{Number of possible connections between these nodes}} $$

whenever the denominator is equal to zero, we impose $C_i = 0$. Note that it follows that $0 < C_i < 1$ for any possible node. Figure 2 illustrates the calculation of the clustering coefficient for a simple network.

**Hierarchical Measurements**: Several complex networks measurements, including the node degree and clustering coefficient, can be generalized to take into account not only the immediate neighborhood of a node, but also those which are at successive distances (i.e. 2, 3, ...) from that specific node. In particular, the hierarchical degree of a node for hierarchical level $i$ corresponds to the number of edges connecting the nodes at distance $i$ to the nodes at distance $i + 1$. The hierarchical clustering coefficient of a given node for hierarchical level $i$ is calculated in the same way as the traditional clustering measurement, but considering the edges between the nodes at distance $i$ and the nodes at distance $i + 1$.

For all measurements, all nodes of the complex network are characterized. To have a global information of the complex network, the mean of a frequency histogram of each measurements is calculated. Two parameters are extracted from these histograms: the mean and the standard deviation.

### 2.2. Image representation

To transform an image into a complex network, we assume that each pixel is represented by one node. Weights of edges are defined by the absolute difference between pixels which they represent. In our case, an edge with weight value equal to zero in matrix $W$ defines no connection, an edge with small value defines a connection with high intensity, and a large value of weight edge defines a poor connection. Connections between edges are defined only inside a circular region of radius $r$ centered on each pixel. An example of construction of the two matrices $W$ and $W_t$ can be seen in Figure 4 and a complete representation in complex networks of image in Figure 5.
Figure 3. Hierarchical representation (b) of the complex network (a).

Figure 4. (a): part of image in Grey level (b): adgency matrix ($W$) of the sub-image representation in complex network (c): $W_t$ matrix with threshold = 8.

Figure 5. (a): A $32 \times 32$ sub-image with the $22 \times 22$ usable centered zone, (b): typical representation of the complex network with threshold = $\pm 2$, (c): representation without the border effect.
3. COMPARATIVES METHODS

The results obtained by using the complex network methodology have been compared to those provided by the co-occurrence matrices introduced by Haralick\(^9\) and by Gabor filters.\(^2,10\)

3.1. Co-occurrence features:

Co-occurrence matrices consider repeated occurrences of some grey level configuration in the texture. A co-occurrence matrix is constructed by observing pairs of pixels separated by a distance \(d\) and incrementing the matrix position corresponding to the grey level of both pixels. The value \(p(i, j)\) represents the frequency of occurrence of the situation \(f(x_1, y_1) = i, f(x_2, y_2) = j, |x_1 - x_2| = d\) or \(|y_1 - y_2| = d\) or \(\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} = d\).

Various characteristics can be extracted from the co-occurrence matrix: Energy, Contrast, Correlation, Dissimilarity, Homogeneity. In our case, these characteristics were determined with \(d = 1\) and \(d = 5\).\(^11\)

3.2. Gabor filters:

Gabor filters which perform a local Fourier analysis, are essentially sine and cosine (complex exponential) modulated by a Gaussian window. In the complex space these filters are expressed as:

\[
h(x, y) = g(x', y').e^{j2\pi(Ux + Vy)}
\]

where \(g(x, y) = \frac{2}{\pi \lambda \sigma^2} e^{-\frac{(x^2 + y^2)}{2\sigma^2}}\) and \[
\begin{bmatrix}
  x' \\
  y'
\end{bmatrix} = \begin{bmatrix}
  x \cos(\phi) + y \sin(\phi) \\
  -x \sin(\phi) + y \cos(\phi)
\end{bmatrix}
\]

\(\phi\) is a clockwise rotation along the \(x\) axis, \(U\) and \(V\) represent the frequency coordinates.

\(\sigma\) is the standard deviation of the Gaussian envelope (which defines its size) and \(\alpha\) is the shape parameter of the Gaussian.

The Gaussian has a circular shape for \(\alpha = 1\).

The transfer function of \(h(x, y)\) is expressed as:

\[
H(x, y) = G(u' - U', v' - U')
\]

with \(G(u, v) = e^{-2\pi^2 \sigma^2(u^2 \lambda^2 + v^2)}\) and \[
\begin{bmatrix}
  u' \\
  v'
\end{bmatrix} = \begin{bmatrix}
  u \cos(\phi) + v \sin(\phi) \\
  -u \sin(\phi) + v \cos(\phi)
\end{bmatrix}
\]

\(H(u, v)\) is therefore a Gaussian band-pass filter, which principal axis is oriented at \(\phi\) degree from the \(u\) axis and with central frequency \(F\) defined by : \(F = (U + V)^{1/2}\) oriented according to the polar angle \(\theta\), as shown in figure 6. In our case, 3 frequency and 6 angles are used, with \(\theta = \phi\). Parameter extracted of this results is the energy:

\[
\text{Energy} = \sum \text{pixel}^2;
\]

4. RESULTS

The comparative study was performed while considering different textures resulting from CUReT * data base. Six different types of textures, illustrated in Figure 7, were used. For all textures, 20 "sub-images" are considered.

Two different classification tests are considered. The first is a multilayer perceptron issue of the software TANAGRA,\(^12\) the second, a Bayesian Classifier. The multilayer perceptron have 25 neurons, with 500 maximum iterations and a learning rate equal to 0.25. The Bayesian Classifier is based on the normal probability model,\(^13\) which equation is given in Equation 5, where \(\mathbf{X}\) is the random vector, \(\mu\) corresponds to the mean vector, and \(K\) is the covariance matrix.

\(^*\)http://www1.cs.columbia.edu/CAVE/software/curet/
Figure 6. General Gabor filter in the Fourier space $\theta \neq \phi$.

Figure 7. Samples of the 6 classes of textures used.

$$p(\mathbf{x}) = \frac{1}{(2\pi)^{N/2} \sqrt{\text{Det}(K)}} \exp \left\{-\frac{1}{2}(\mathbf{x} - \mathbf{\mu}_X)^T K^{-1}(\mathbf{x} - \mathbf{\mu}_X) \right\}$$

(5)

The two methods use 50% of "sub-images" for training and 50% for classification. To compare our results of classification with comparative’s method, the Error Rate is defined as the number of bad recognitions divided by the number of samples.

4.1. Traditionnal measurements

Table 1 shows error rate of classification for our method (with different thresholds), and for comparison, with the results obtained by using the co-occurrence matrices and Gabor Filters.

<table>
<thead>
<tr>
<th>Method</th>
<th>distance r</th>
<th>threshold</th>
<th>E.R. perceptron</th>
<th>E.R. Bayesian</th>
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</thead>
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<td>2</td>
<td>0.19</td>
<td>0.10</td>
</tr>
<tr>
<td>Graph</td>
<td>5</td>
<td>5</td>
<td>0.12</td>
<td>0.13</td>
</tr>
<tr>
<td>Haralick</td>
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<td>/</td>
<td>0.23</td>
<td>0.17</td>
</tr>
<tr>
<td>Haralick</td>
<td>5</td>
<td>/</td>
<td>0.26</td>
<td>0.17</td>
</tr>
<tr>
<td>Gabor</td>
<td>/</td>
<td>/</td>
<td>0.23</td>
<td>0.17</td>
</tr>
</tbody>
</table>

Table 1. Error Rate (E.R.) for our simple method, Haralick’s approach and Gabor filters.

4.2. Hierarchical measurements

The error rate is also determined with all different hierarchical levels. The evolution of this error in terms of the consideration of successive hierarchical levels ($x$–axis) can be seen in Figure 8. Note that each value $k$ along the $x$–axis indicates the use of all hierarchical levels up to $k$, and not just the hierarchical level $k$.

With these results it appears that the use of measurements considering progressive hierarchical levels has a definite effect in improving the classification rate (lower error rate). The minimum of Error Rate is not obtained for the highest hierarchical levels used (3 and 4). This better hierarchical level depends of the image size, the circular region of definition of connection, the threshold. If the hierarchical level is too high, connections between
Figure 8. Evolution of Error Rate in function of hierarchical measurements used. Results of classification is obtained by the multilayer perceptron.

nodes are poor (or non-existent), and thus parameters determined (degree and clustering coefficient) can not be good to discriminate classes of networks.

5. CONCLUSION

Two methods for texture classification using complex networks had been presented and compared. Our simple method, using Complex networks with measurements of topology and connectivity, has a good ability to represent and characterize textures. The interest of hierarchical levels was made and increase the efficiency of the classification of characterisation of textures. Although promising results have been obtained, our method used simples parameters (mean and standard deviation). We are currently working to improve this shortcoming: use more informations about the histograms (i.e. moments and coefficients), an automatic determination of the better hierarchical level to improve the classification.

REFERENCES
