Image Processing With A Cellular Nonlinear Network.

S.Morfu

Laboratoire d'Electronique, Informatique et Image (LE2i) UMR C.N.R.S 5158, Aile des Sciences de l’Ingénieur, BP 47870, 21078 Dijon Cedex, France

Abstract

A Cellular Nonlinear Network (CNN) based on uncoupled nonlinear oscillators is proposed for image processing purposes. It is shown theoretically and numerically that the contrast of an image loaded at the nodes of the CNN is strongly enhanced, even if this one is initially weak. An image inversion can be also obtained without reconfiguration of the network whereas a gray levels extraction can be performed with an additional threshold filtering. Lastly, an electronic implementation of this CNN is presented.

Key words: Nonlinear oscillator, Cellular Nonlinear Network, Image processing.

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1 Introduction

Since the pioneer work of L. Chua concerning Cellular Nonlinear Networks (CNN) [1], it has become clear that nonlinear media can be considered as parallel multiprocessors systems devoted to signal processing. Indeed, their efficiency has been widely used to solve problems of high computational complexity, like for instance, finding the optimal path in a two dimensional vector field [2] or finding the shortest path in a labyrinth [3]. The efficiency of nonlinear signal processing comes from an additional dimension lying in the signal amplitude, which gives rise to new properties not shared by linear system. Noise filtering with a nonlinear dissipative lattice [4], image processing with nonlinear networks [1,5–7], or signal detection/transmission via the nonlinear stochastic resonance phenomenon [8–12], are few examples of a nonrestrictive list, where taking into account nonlinearity allows to transcend the limitation

Email address: smorfu@u-bourgogne.fr (S.Morfu).

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of classical linear processes.

In many recognition problems, basic image processing tasks are also performed with nonlinear systems. One can cite edge detection with an equivalent Nagumo electrical network [13,14], construction of image skeleton using reaction diffusion processor realized with chemical nonlinear media [15], or restoration of individual components with overlapped components [16]. It is important to note that all these basic tasks can be realized in good conditions only if the initial image is sufficiently contrasted.

The aim of this paper is to propose a new Cellular Nonlinear Network (CNN), built with uncoupled oscillators, which mainly allows the contrast enhancement and the video inversion at two closely different processing times, without needing a reconfiguration of the network.

The paper is organized as follow. In section II, using the mechanical analogy of a particle experiencing a double well potential, we first investigate the nonlinear oscillators dynamics. Especially, it is theoretically shown that two particles with very close initial conditions can present, as time goes on, a maximum, a minimum or a null difference of amplitude, which may constitute a contrast enhancement of the weak initial difference of amplitude. Sect. III, is devoted to present applications of the properties exhibited in Sect. II to a bi-dimensional signal like the weak contrasted picture of figure 1. Each pixel corresponding to a nonlinear oscillator, a contrast enhancement with or without image inversion is obtained at two closely different times, while a gray level extraction is possible with an appropriate threshold filtering. In section. IV, an electronic design of the CNN is proposed and an experimental characterization of the elementary cell of the CNN is reported. We finally conclude with a discussion to propose further studies and applications.

2 Theoretical study: Nonlinear oscillator properties.

In this section, we briefly introduce the properties of the oscillators used to perform different image processing tasks.

The equation describing the motion of a particle submitted to a nonlinear force \( f(U) = -\omega_0^2(U - m)(U - m - \alpha)(U - m + \alpha) \) deriving from the double well potential of figure 2 is given by

\[
\frac{d^2 U}{dt^2} = f(U), \tag{1}
\]

where \( U \) represents the particle displacement, while the parameter \( \alpha < m \) of the nonlinear force adjusts the potential barrier height \( E_b = \omega_0^2 \alpha^4/4 \).

Setting \( x = U - m \), eq. (1) can be normalized as
\[
\frac{d^2x}{dt^2} = -\omega_0^2 x(x - \alpha)(x + \alpha). \tag{2}
\]

Solutions of eq.(2) for a zero initial velocity are given by the following Jacobian elliptic functions:

\[x(t) = x_0 \, cn(\omega t, k), \tag{3}\]

where \(x_0\), \(\omega\) and \(0 \leq k \leq 1\), correspond respectively to the oscillations amplitude, the pulsation and the modulus of the Jacobian elliptic function \(cn\).

Using the properties [17] of the \(cn\) function and deriving twice equation (3), we get

\[
\frac{d^2x}{dt^2} = -2 \frac{k \omega^2}{x_0^2} x [x^2 - \frac{2k-1}{2k} x_0^2], \tag{4}
\]

which provides, after identification with eq.(2), the pulsation \(\omega\) of the \(cn\) function

\[\omega(x_0) = \omega_0 \sqrt{x_0^2 - \alpha^2}, \tag{5}\]

and its modulus

\[k(x_0) = \frac{1}{2} \frac{x_0^2}{x_0^2 - \alpha^2}. \tag{6}\]

Writing the initial condition \(U^0 = x_0 + m\), solutions of eq. (1) can be straightforwardly deduced from eqs. (3), (5), (6), that is

\[U(t) = m + (U^0 - m) \, cn(\omega t, k), \tag{7}\]

with

\[
\omega(U^0) = \omega_0 \sqrt{(U^0 - m)^2 - \alpha^2}, \tag{8}
\]

\[k(U^0) = \frac{1}{2} \frac{(U^0 - m)^2}{(U^0 - m)^2 - \alpha^2}. \tag{9}\]

Both parameters \(\omega\) and \(k\) appear then as driven by the amplitude of the solution, that is the initial condition \(U^0\).

Exressing the constraints \(|U^0 - m| > \alpha\) for eq. (8) and \(0 \leq k \leq 1\) for eq. (9), we deduce the allowed range of the initial condition \(|U^0 - m| \geq \alpha \sqrt{2}\) which corresponds to a particle with an initial potential energy above the energy barrier \(E_B\) (see on fig.2 points \(O\) and \(O'\) located at \(U^0 = m - \alpha \sqrt{2}\) and
Now, we investigate the dynamics of two independent oscillators $O_1$ and $O_2$ starting at the even time, but with a very weak difference of initial condition, namely $\epsilon$.

If we note the initial oscillators amplitudes $U_0^1$, $U_0^2$ and their respective displacement $U_1$, $U_2$, then their pulsation and modulus are driven by their respective initial amplitude according to eq. (8) and eq. (9). In order to illustrate the oscillators properties, we have represented figures 3.a and 3.b respectively, the position time dependance of the two oscillators $O_1$ and $O_2$, and their displacement difference $\delta$ in the case $\alpha = 0.937$, $m = 2.607$, $U_0^1 = 0.423$, $U_0^2 = 0.677$, $\omega_0 = 1/\sqrt{10^{-3}} \times 100 \times 10^{-9}$. Note that we choose this set of parameters to allow, in section 4, a direct comparison between the theoretical model and its electronic implementation.

In Fig. 3.a, the oscillations take place in the range $[0.423; 4.791]$ for the first oscillator and in $[0.677; 4.537]$ for the second one, which is in agreement with the range $[U_0^i; 2m-U_0^i] i \in \{1,2\}$ deduced from eq. (7). Moreover, the two oscillators quickly achieve a phase opposition when $U_2 = 4.537$ and $U_1 = 0.423$, inducing a maximum displacement difference $\delta = 4.537 - 0.423 = 4.114$ at the time $t_{opt} = 119.4 \times 10^{-6}$ (dotted line in figure 3.a and b). The weak initial displacement difference $\epsilon = U_0^2 - U_0^1 = 0.254$ is then strongly increased at the time $t_{opt} = 119.4 \times 10^{-6}$, which constitutes a contrast enhancement.

To valid the theoretical study, we have directly integrated eq. (1) with a fourth order Runge-Kutta algorithm for the two different initial conditions $U_0^1$ and $U_0^2$. In figure 3.b, the numerical results (• signs) match with a perfect agreement the analytical expression of the displacement difference (solid line) and confirm the previous properties of nonlinear oscillators.

Finally, to bring to the fore the genuine interest of nonlinear systems, we consider two independent oscillators submitted to a linear force $f(U) = -\omega_0^2 U$, starting at the even time and with a weak difference of initial condition $\epsilon$. In this linear case, the oscillators relative displacement obeys to

$$U_2 - U_1 = \epsilon \cos(\omega_0 t),$$

and always remains in the range $[-\epsilon, \epsilon]$. Therefore, linear systems are unable to enhance a weak amplitude contrast, contrary to nonlinear systems which perform efficiently this task thanks to an additional dimension lying in the initial amplitude $U_0^0$.
In this section, we propose 3 image processing tools inspired by the properties of nonlinear oscillators. By analogy with a particle experiencing a double well potential, the pixel number \((i, j)\) is analog to a particle (oscillator) whose initial position corresponds to the initial gray level \(X_{i,j}^0\) of this pixel. Therefore, we are led to investigate the dynamics of a CNN realized with uncoupled nonlinear oscillators. The image to process is first loaded as initial condition at the nodes of the CNN. Then, for a processing time \(t\), the gray level of each pixel is obtained noting the position reached by the corresponding oscillator at this time \(t\).

### 3.1 The differential contrast

To take into account the standard coding of image which involves a pixel dynamics in the range \([0; 1]\), we restrict the parameters of the nonlinearity to \(m = 1/2\) and \(\alpha < 1/2\). Under these conditions, if \(X_{i,j}\) denotes the gray level of the pixel number \((i, j)\) and \(N \times M\) the image size, the set of equations modelling a network of \(N \times M\) decoupled particles submitted to a force deriving from the double well potentials of figure 4.(a) reduces to

\[
\frac{d^2X_{i,j}}{dt^2} = -\omega_0^2 \left( X_{i,j} - \frac{1}{2} \right) \left( X_{i,j} - \frac{1}{2} - \alpha \right) \left( X_{i,j} - \frac{1}{2} + \alpha \right)
\]

\[
with \ i = 1, 2..N, \ j = 1, 2, ..M , \quad (11)
\]

where all gray levels of the initial image are in \([0 ; 1/2 - \alpha \sqrt{2}]\) to ensure an initial potential energy above the barrier.

In order to describe the dynamics of the CNN, we will mainly consider the time evolution of two pixels corresponding initially to the minimum and maximum gray level of the weak contrasted picture. If \((i_1, j_1)\) and \((i_2, j_2)\) represent their respective coordinates, then their gray level difference \(\Delta(t) = X_{i_2,j_2}(t) - X_{i_1,j_1}(t)\)-called Differential Contrast in the whole article-characterizes at \(t = 0\) the weak contrast of the image to process (For instance, \(\Delta(t = 0) = 0.05\) for the image of figure 1).

In Figure 4.(b), the plot of this Contrast \(\Delta(t)\) presents a periodic behavior with local minima and maxima, revealing the possibility to realize a contrast enhancement.

Indeed, the Differential Contrast can be null-for \(t=3.74\) and \(t=7.31\) for instance-or it can reach local minima-namely for \(t=2.87\) and \(t=16.33\)-., else it can achieve
local maxima—\( t = 6.24, t = 12.82 \), to cite a few. The maximum value of the Differential Contrast is obtained at \( t_{\text{opt}} = 19.91 \) and is, according to the theoretical section 2, \( \Delta(t_{\text{opt}} = 19.91) = 0.95 \).

### 3.2 Contrast enhancement and image inversion

To investigate the CNN response, we have numerically integrated eq. (11) loading the image of figure 1 as initial condition. Then, for the characteristic times of the previous subsection, we have reported the resulting images in figure 5.

The analytical expression of the contrast \( \Delta(t) \), which can be deduced from eq. (7-9) setting \( m = 1/2 \) and \( U = X_{i,j} \), allows to explain the temporal evolution of the initial image in the \( CNN \). Indeed, the image goes through local contrast minima defined by the zeros of the Differential Contrast \( \Delta(t) \) (for \( t = 3.74 \) and \( t = 7.31 \)), as represented in figure 5.(b) and 5.(d).

On the other hand, the local maxima of \( \Delta \) correspond to local contrast enhancements of the initial picture with a growing quality versus time (figures 5.(c) 5.(e) and 5.(g) for \( t = 6.24, t = 12.82 \) and \( t = 19.91 \) respectively). A contrast optimum for a processing time \( t_{\text{opt}} = 19.91 \) is then reached (figure 5.g) as predicted by figure 4.(b) (dotted lines).

The local minima, achieved namely for processing times \( t = 2.87 \) and \( t = 16.33 \), give also good contrast enhancements (figure 5.(a) and 5.(f)). However, the resulting images are inverted since the minima of the Differential Contrast are negative (see figure 4.(b)). This is the main property of the system: at two closely different processing times, it is possible without reconfiguration of the oscillators network to obtain a contrast enhancement with or without image inversion (fig 5.(f) for \( t = 16.33 \) and 5.(g) for \( t = 19.91 \)).

Note that the histogram of each image shows clearly the oscillator network dynamics. Indeed, from an histogram of a weak contrasted image (figure 1), where all gray levels are located in a small range around 0, the histogram evolution versus time reveals that the range of gray levels periodically increases (figures 5.(a),(c),(e)) and decreases (figures 5.(b) and 5.(d)) versus time, conveying that the picture goes periodically from a weak contrasted situation to a higher contrasted one.

### 3.3 The contrast curves

To characterize the realized contrast, we define a contrast curve as the plot of the pixels gray level of the processed image versus their initial gray level. The horizontal axis corresponds to the initial gray scale ([0; 0.05] in our case), whereas the vertical axis represents the gray scale of the processed image.
Since the contrast enhancement with the largest gray scale is performed for the time $t_{opt}$, the contrast curve can then be adjusted with the parameter $\alpha$ of the nonlinearity for the corresponding time $t_{opt}$, that is changing the width and height of the potential barrier. For each values of $\alpha$, we have first determined the corresponding optimum processing time $t_{opt}$. Then, we have performed a numerical simulation for this specific couple $(\alpha, t_{opt})$ to plot the resulting contrast curve. Figure 6 summarizes the system response for three different values of the parameters couple $(\alpha, t_{opt})$. To compare our contrast enhancement to a uniform one, we have superimposed in dashed line the contrast curve obtained with a change of gray scale, namely multiplying the initial gray scale $[0, 0.05]$ by a scale factor.

The first curve (fig. 6.a) obtained for the greatest value of $\alpha$, that is with the greatest potential barrier, reveals that the contrast of the medium gray levels are unchanged compare to a uniform processing (dashed line) whereas the contrast of the dark and light grays are strongly (but not symmetrically) enhanced with a distortion in their extreme gray values. This slight distortion only occurs when the gray levels of the image to process are launched with an energy near the energy barrier $E_b$, which is the case for $\alpha = 0.3$.

By contrast, for $\alpha = 0.25$, the light and dark gray are quasi symmetrically enhanced (figure 6.b), while increasing $\alpha$ involves a brighter resulting images (see on figure 6.c the relative position of the contrast curves compare to the dashed line).

### 3.4 Gray level extraction

Here, we propose a gray level extraction realized with this nonlinear system for processing times exceeding $t_{opt}$. In a sake of clarity, we will rather consider the image of figure 7.(a) consisting of a continuous gray scale. Once the initial gray scale divided by 20 (to be in the previous range $[0, 0.05]$), the gray levels of each pixel are loaded as initial conditions in the nonlinear oscillators network, which provides, for different processing times beyond $t_{opt}$, the contrast curves presented in figure 7.

The evolution of these curves versus time reveals that their minimum is successively reached for each level of the initial gray scale. It means that the pixels with the same initial gray level, take, after a given processing time, the minimal gray level of the processed image. Therefore, time acting as a discriminating parameter, an appropriate threshold filtering allows to extract all pixels in a given range of gray level.

Indeed, the simple case of a constant threshold versus time $V_{th} = 0.055$ is reported in figure 7 where 8 gray ranges are extracted from the initial image at 8 closely different processing times. As exhibited in figure 7, the width of these ranges, which depends on the difference of the threshold and the minimum of the contrast curves, reduces in the light grays.
Of course, to ensure a quasi constant width for all extracted ranges of gray, the threshold \( V_{th} \) has to follow, with a slight offset, the temporal evolution of the contrast curves minimum. Under this condition, this offset value adjusts the width of the extracted gray range. Especially, a null offset allows to separate all gray levels, which realizes a gray level extraction with a perfect efficiency. Note that the evolution of the threshold can be deduced with the theoretical expressions (7-9) determining at each processing time the minimum of the corresponding contrast curve.

4 The electronic implementation of the CNN

In this last section, we propose an analog electrical lattice based on the properties of (1) that could realize in real time a contrast enhancement of an image loaded at the nodes of the lattice. This Cellular Nonlinear Network (CNN) is constructed with \( N \times M \) decoupled elementary cells. To obtain the cubic nonlinearity of eq. (1), we first realize a polynomial source \( P(U) = U - (U - m_1)(U - m_2)(U - m_3) \) with analog multipliers AD633JN and classical operational amplifier (TL081CN) allowing to balance the scale factor \( 1/10V^{-1} \) of the multipliers (figure 8). The worth of the zeros \( m_1, m_2, m_3 \) is adjusted with external voltage sources to \( m_1 = m - \alpha, m_2 = m, m_3 = m + \alpha \), which ensures a strict homogeneity in the CNN.

Then, each elementary cell of the CNN includes such a polynomial source and a linear inductance assuming a feedback between the input/output of the polynomial source (Figure 9). The potential difference between the input/output source of the cells \((i,j)\) of the network obeys to

\[
U_{i,j} - P(U_{i,j}) = (U_{i,j} - m)(U_{i,j} - m - \alpha)(U_{i,j} - m + \alpha),
\]

and is the opposite of the nonlinearity \( f(U_{i,j}) \) of eq. (1).

As represented in figure 10, a perfect agreement between the cubic law (solid line) and the experimental characteristic (+ signs) is observed.

Owing to the high input impedance of the multipliers (10MΩ), the current \( I \) in the branch of the linear inductance \( L \) is also present in the branch of the linear capacitor \( C \), involving:

\[
I = C \frac{dU_{i,j}}{dt}
\]

\[
L \frac{dI}{dt} = P(U_{i,j}) - U_{i,j}.
\]
Deriving eq. (13) and substituting into (14), the differential equation describing the evolution of the voltage $U_{i,j}$ at the node $(i,j)$ of the CNN reduces to:

$$\frac{d^2 U_{i,j}}{dt^2} = -\frac{1}{LC}(U_{i,j} - m)(U_{i,j} - m - \alpha)(U_{i,j} - m + \alpha).$$

(15)

Eq. (15) appears then as an analog simulation of equation (1) with $\omega_0 = \frac{1}{\sqrt{LC}}$. The time evolution of two cells with a slight difference of initial conditions (namely $0.677V$ and $0.423V$ like in the theoretical section 2) is represented figure 11.a and b respectively.

In a relative good agreement with the oscillation range $[U^0; 2m - U^0]$ established in the section 2, the oscillations take places between $U^0 = 0.677V$ and $4.48V$ for the oscillator of fig. 11.a, and in $[0.423V, 4.76V]$ for the oscillator of fig. 11.b. Moreover, the main property of nonlinear oscillators is verified since the oscillation pulsation is driven by the initial condition. Indeed, a phase opposition, corresponding to the case $U_2 = 4.48V$ and $U_1 = 0.423V$ (dotted line of figures 11.a and 11.b respectively), is quickly reached at $t_{opt} = 124.5\mu s$, which gives a strong contrast enhancement in figure 11.c at $t_{opt}$, namely 4.057V. Despite a slight discrepancy between the experimental and theoretical optimal processing time (less than 5% and mainly imputable to the component uncertainties), a fairly good contrast enhancement is achieved. Note that the experimental processing time to obtain the best contrast enhancement can then be adjusted with the worths of $L$ and $C$ to match real time processing constraints.

Lastly, to implement a gray level extraction, classic electronic comparators can be added at each node of the C.N.N. The main constraint to perform a perfect gray level detection lies in the threshold value. Indeed, the threshold have to follow the temporal evolution of the contrast curves minimum, as stated in the section 3.4.

5 Conclusion

In this paper, a new cellular nonlinear network has been proposed to perform basic image processing tasks, such as contrast enhancement, image inversion. The main advantage of the system is to realize all these operations without needing a network reconfiguration. Especially, we have shown that the shape of the nonlinearity affects the quality of the resulting contrast. Consequently, changing the nonlinearity could provide a rich variety of contrast curves. Furthermore, using the processing time as a discriminating parameter, we have also reported the possibility, for this CNN, to perform a gray level extraction with a resolution which can be tuned by a threshold parameter. An electronic
sketch of this CNN is finally presented for possible experimentations in real
time image processing.
One might think that adding a linear or nonlinear coupling between the os-
cillators could allow to implement other tasks currently performed in image
processing. Therefore, this CNN constitutes a framework for further investi-
gations in nonlinear signal and image processing.

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**Figure captions**

**Fig. 1:** Weak contrasted image of Lena and its histogram. Image minimum $X_{i_1,j_1}^0 = 0$ located at pixel $(i_1, j_1) = (111,92)$, image maximum $X_{i_2,j_2}^0 = 0.05$ located at pixel $(i_2, j_2) = (69,30)$. Image size $128 \times 128$ pixels.

**Fig. 2:** Double well potential represented for $\alpha = 0.937$ and $m = 2.607$, in which a particle evolves from an arbitrary initial position $U^0 < m - \alpha \sqrt{2}$, that is with an initial potential energy above the barrier $E_b$.

**Fig. 3:** (a) Displacement $U_1$ (continuous line) and $U_2$ (dashed line) versus time of the two respective oscillators $O_1$ and $O_2$ obtained with expressions (7-9). (b) Displacement difference or contrast amplitude $\delta = U_2 - U_1$ of the two oscillators versus time (Solid line: theoretical results; •: numerical results). Parameters: $U_1^0 = 0.423$, $U_2^0 = 0.677$, $\alpha = 0.937$, $m = 2.607$, $\omega_0 = 1/\sqrt{10^{-3} \times 100 \times 10^{-9}}$.

**Fig. 4:** (a): Double well potential suitable for image processing represented for $\omega_0 = 2\sqrt{2}$ and $\alpha = 0.25$ inducing a gray level dynamics in the range $[0; \ 1]$. (b): Differential Contrast $\Delta(t)$ defined as the oscillation difference between the maximum and minimum gray levels of the initial image. Parameters: $\omega_0 = 2\sqrt{2}$, $\alpha = 0.25$, $X_{i_1,j_1}^0 = 0$, $X_{i_2,j_2}^0 = 0.05$, that is $\Delta(t=0) = 0.05$.

**Fig. 5:** Images and their histogram for different processing times. Parameters: $\alpha = 0.25$, $\omega_0 = 2\sqrt{2}$: (a) : $t = 2.87$, (b) : $t = 3.74$, (c) : $t = 6.24$, (d) : $t = 7.31$, (e) : $t = 12.82$, (f) : $t = 16.33$, (g) : $t = 19.91$. (b) and (d) represent local minima of contrast provided by the zeros of $\Delta$. (c), (e) and (g), corresponding to contrast enhancements, are obtained with the local maxima of $\Delta$ whereas the minimum of $\Delta$, images (a) and (f), allow a contrast enhancement with image inversion.

**Fig. 6:** Contrast enhancement versus the nonlinearity parameter $\alpha$ for the corresponding optimal processing time $t_{opt}$ (solid line) compared to a uniform enhancement (dashed line). (a): $(\alpha, t_{opt}) = (0.3, 15.9)$; (b): $(\alpha, t_{opt}) = (0.25, 19.91)$; (c): $(\alpha, t_{opt}) = (0.05, 26.4)$. Parameter: $\omega_0 = 2\sqrt{2}$.

**Fig. 7:** Gray levels extraction. (a) image to process consisting of a continuous gray scale. Contrast curves obtained for processing times exceeding $t_{opt} = 19.91$, namely (b) : $t = 26.45$, (c) : $t = 27.17$, (d) : $t = 27.87$, (e) : $t = 28.65$, (f) : $t = 29.43$, (g) : $t = 30.29$, (h) : $t = 31.22$ and (i) : $t = 31.85$. Parameters:
\( \alpha = 0.25, \omega_0 = 2\sqrt{2} \). At the top of each contrast curve, the resulting image is then threshold filtered replacing the pixel gray level with one (white) if that gray level exceeds the threshold \( V_{th} = 0.055 \) (dotted line), otherwise with 0 (black).

**Fig. 8:**

Polynomial source generation. The triangle represents a classical inverter amplifier with \(-10\) amplification.

**Fig. 9:** Sketch of the electronic elementary cell number \((i, j)\) of the CNN. The initial condition \( U_{i,j}^0 \) is introduced via a 1N4148 diode adding an offset equal to the diode threshold \( V_T \).

**Fig. 10:** Potential difference between the input/output source of the cell \((i, j)\). Solid line: cubic expression (12) of the nonlinearity \(-f(U)\); (+) signs: experimental results. Parameters: \( \alpha = 0.937V, m = 2.607V \).

**Fig. 11:** Experimental enhancement of a weak difference of initial conditions. Parameters: \( L = 1mH, C = 100nF, \alpha = 0.937V, m = 2.607V \). (a and b) Time evolution of two cells with respective initial conditions 0.677V and 0.423V. (c) Time evolution of the voltage difference between the two cells.
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