Denoising 3D Models with Attributes using Soft Thresholding

Michaël Roy*, Sebti Foufou, and Frédéric Truchetet

Le2i - CNRS UMR 5158- Université de Bourgogne
12 rue de la fonderie - 71200 Le Creusot, France

ABSTRACT

Recent advances in scanning and acquisition technologies allow the construction of complex models from real world scenes. However, the data of those models are generally corrupted by measurement errors. This paper describes an efficient single pass algorithm for denoising irregular meshes of scanned 3D model surfaces. In this algorithm, the frequency content of the model is assessed by a multiresolution analysis that requires only 1-ring neighbourhood without any particular parameterization of the model faces. Denoising is achieved by applying the soft thresholding method to the detail coefficients given by the multiresolution analysis. Our method is suitable for irregular meshes with appearance attributes such as normal vectors and colors. Some results of real world scene models denoised with the proposed algorithm are given to demonstrate its efficiency.

Keywords: Denoising, irregular mesh, multiresolution analysis, surface attributes, soft thresholding.

1. INTRODUCTION

With each year, the cost of range scanners continues to drop while their accuracy continues to improve. Although the fidelity of these scanners has improved dramatically over the past decade, noise is ever present in any practical system. Reconstruction algorithms associated with these technologies typically output triangular meshes due to their simplicity and versatility in representing a variety of shapes and topologies. However, raw data produced by 3D scanners are corrupted by measurement errors. Errors in measurement and registration manifest as noise in the geometry of the mesh. For example, range imaging in computer vision samples the surfaces of a scene and creates point-cloud models. The precision in estimating the position of these points is a function of the sensor mechanics, instrument electronics, surface orientations, and reflective properties. With the variability among these elements, measurement error is inevitable. Our goal is to remove the geometric and attribute noise of a surface mesh while keeping the features of the surface itself.

In this paper, we propose a denoising algorithm for irregular triangular meshes of scanned 3D real world models containing appearance attributes such as colors or normal vectors. In this work, we use a multiresolution technique to construct an efficient denoising algorithm. This technique allows one to obtain the mesh frequency content and a wavelet shrinkage method operates on the frequency content to denoise the mesh.

Multiresolution analysis has recently proved its efficiency for irregularly sampled meshes. The analysis decomposes a mesh into coarser meshes representing the initial model at different scales. A set of detail coefficients are computed for each scale, defining the frequency content of the model. The multiresolution analysis proposed in this paper is computationaly simple, and adapted for irregularly sampled meshes.

We take advantage of the multiresolution analysis to build a denoising filter suitable for irregular meshes. The denoising algorithm estimates the variance of the detail coefficients and modifies them with the soft thresholding technique. Results show the efficiency of the overall method on 3D scanned models.

Section 2 presents the background and related work on this topic. In Section 3 we give an overview of the multiresolution analysis designed for irregular meshes. Section 4 introduces the denoising method used with the multiresolution analysis. Section 5 shows some experimental results. Conclusion and ideas for future work are given in Section 6.

* E-mail: michael.roy@iut-dijon.u-bourgogne.fr

2. RELATED WORK

In the following, we discuss the related work on multiresolution analysis of meshes and surface denoising.

2.1. Multiresolution Analysis of Meshes

Multiresolution analyses based on wavelet theory have started a new research domain on hierarchical methods for computer graphics.[1] Lounsbery[2] made the connection between wavelets and subdivision to define different levels of resolution. This technique called Subdivision Wavelet Transform makes use of the theory of the multiresolution analysis and of the subdivision rules to construct a multiresolution surface representation for surface with subdivision connectivity. Zorin et al.[3] proposed a combination of subdivision and smoothing algorithms to construct a set of algorithms for interactive multiresolution editing of complex meshes with arbitrary topology. The authors used Loop subdivision for the estimation of the high resolution mesh from the coarse representation.

Those methods assume that the mesh is semi-regular and use traditional subdivision schemes. More recently, different approaches have been presented in order to deal with irregular meshes. Bonneau[4] introduced the concept of multiresolution analysis over non-nested spaces, which are generated by the so-called BLaC-wavelets, a combination of the Haar function with the linear B-Spline function. This concept was then used to contruct a multiresolution analysis over irregular meshes. Kobbelt et al.[5] proposed a multiresolution editing tool for irregular meshes using the discrete fairing method. The authors use the progressive mesh algorithm[6] to build the coarse resolution mesh. A smoothing operator is used to estimate the high resolution mesh. Guskov et al.[7] presented a series of non-uniform signal processing algorithms designed for irregular triangulation. They used a smoothing algorithm combined with existing hierarchical methods to build subdivision, pyramid, and wavelet algorithms for irregular connectivity meshes. The authors proposed a non-uniform subdivision to build a geometrically smooth mesh with the same connectivity as the original mesh. Recently, Valette et al.[8] presented a new wavelet-based multiresolution analysis of irregular surface meshes. This method is based on Lounsbery decomposition. The authors proposed a new irregular subdivision scheme, which allows the algorithm to be applied directly to irregular meshes. They use three codebooks to describe the different merge-split cases, and define some contraints in order to keep the simplification/decomposition step reversible. The method is a fine-to-coarse decomposition, and use a complex simplification algorithm in order to define surface patches suitable for the irregular subdivision. It might be possible to include the attributes in this analysis method, but this has not been done yet.

The multresolution mesh analysis framework used in this paper is based on Kobbelt[5] and Guskov[7] decompositions. The power of these decompositions has been demonstrated through a number of applications including filtering, editing and texture mapping. Our framework created a discrete number of levels of detail, which allows more efficient surface processing compared to continuous levels of detail. Our method can also handle attributes such as colors, normals, and texture coordinates, which gives an multiresolution analysis of a complete model.

2.2. Surface Denoising

Peng *et al.*[9] developed a denoising technique for semi-regular meshes based on locally adaptative Wiener filtering. The subbands of a multiscale representation are modeled as a product of a Gaussian random vector with a hidden multiplier. Estimation of the multiplier leads to the estimation of the local variance and allows standard Wiener denoising. The resulting algorithm is quite efficient and requires only a single pass over the surface at each resolution level. Alexa[10] has proposed a Wiener filtering method for irregular meshes. He has defined a local autocorrelation allowing the design of filters. Unfortunately, this method requires the solution of a linear system. Other methods, known as smoothing or fairing, have been developed in the literature.[11–15] These methods use deterministic approaches and are different from the statistical approach proposed in this paper. Moreover none of them is able to manage appearance attributes such as colors or normal vectors.

3. MULTIRESOLUTION ANALYSIS

We have presented previously a multiresolution analysis framework for irregular meshes with appearance attributes.[16] Multiresolution analysis provides a framework that rigorously defines various approximations and fast analysis algorithms. This framework iteratively contructs approximation and detail parts, forming levels of resolution of the original data set. The details capture the local frequency content of the data set and are used to exactly reconstruct the original data set.

In this section, we briefly present the previously introduced scheme for multiresolution analysis of irregular meshes.

3.1. Decomposition

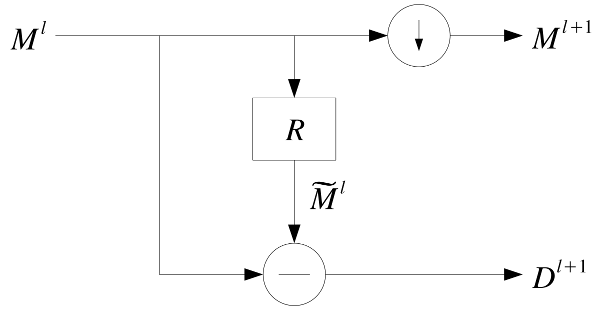


Figure 1. Decomposition scheme of the multiresolution analysis framework for irregular meshes with attributes.

The multiresolution decomposition scheme is presented in Figure 1. The decomposition can be done in three steps:

1. Surface attribute relaxation
2. Detail coefficient computation
3. Mesh dowsampling

The first step of the decomposition is the computation of the relaxed attributes of every vertex of the input mesh. Then the detail coefficients are computed as the difference between the attributes of the vertices of the input mesh and the attributes of the vertices of the relaxed mesh. Finally the mesh is downsampled to create a coarse approximation mesh. The downsampling algorithm creates two sets of vertices: the odd vertices O^l that are removed, and the even vertices E^l that remain to create the coarse approximation. Not that the even vertices are not changed during the decomposition process, and the coarse approximation is a subsampled version of the fine mesh.

A mesh is represented as a couple $M = (V, K)$ where V is a set of vertices representing the geometry and the appearance attributes of the mesh, and where K is a simplicial complex representing the topology of the mesh. Starting with the relaxation step, all the attributes $\mathbf{a}^l_{i,r}$ of the vertices v^l_i of the mesh $M^l = (V^l, K^l)$ at the scale level l are relaxed by the relaxation box R to form a new relaxed mesh $\widetilde{M}^l = (\widetilde{V}^l, K^l)$:

$$\tilde{\mathbf{a}}^l_{i,r} = \mathbf{R}(\mathbf{a}^l_{i,r}). \tag{1}$$

The new relaxed mesh $\widetilde{M}^l$ is composed of the relaxed attributes $\tilde{\mathbf{a}}^l_{i,r}$ of the new vertices $\tilde{v}^l_i \in \widetilde{V}^l$. The relaxation is done using the following equation:

$$\mathbf{R}(\mathbf{a}^l_{i,r}) = \sum_{j \in N^l_1(i)} \lambda^l_{i,j}.\mathbf{a}^l_{j,r}, \tag{2}$$

$$\lambda^l_{i,j} = \frac{\cot \alpha^l_{i,j} + \cot \beta^l_{i,j}}{\sum_{k \in N^l_1(i)} \cot \alpha^l_{i,k} + \cot \beta^l_{i,k}}, \tag{3}$$

where $N^l_1(i)$ represents the 1-ring neighborhood of the vertex i at the level l. The $\lambda^l_{i,j}$ are weights of the relaxation operator minimizing the curvature energy of an edge $e_{i,j}$ where $\alpha^l_{i,j}$ and $\beta^l_{i,j}$ are the angles opposite to the edge $e_{i,j}$ (see Figure 2). It is important to store the $\lambda_{i,j}$ weights to be re-used for the reconstuction.

The detail coefficients $D^{l+1} = \{\mathbf{d}^{l+1}_i\}$ are computed as the difference between the attributes of the input mesh and the attributes of the relaxed mesh:

$$\mathbf{d}^{l+1}_{i,r} = \mathbf{a}^l_{i,r} - \tilde{\mathbf{a}}^l_{i,r}. \tag{4}$$

The final step is the downsampling of the mesh. During this step the vertices to be removed are designated using the global downsampling method. This technique was presented by Kobbelt[5] in order to achieve optimal performance with

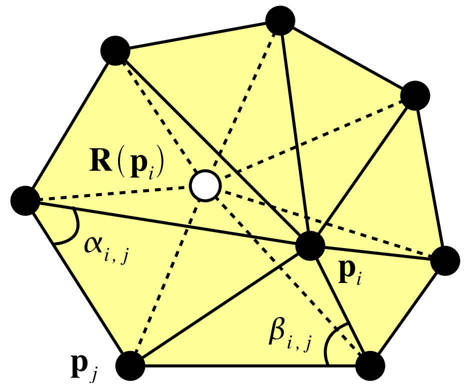
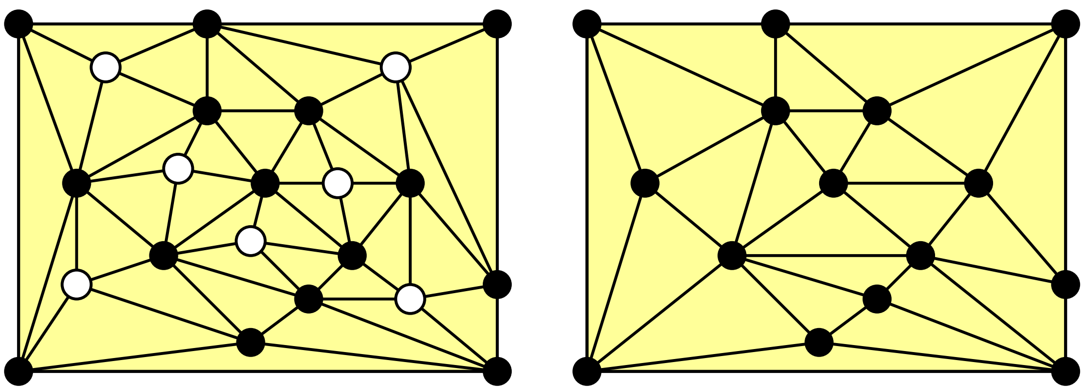


Figure 2. Relaxation of a vertex $\mathbf{p}_i$ according to the local neighborhood.



(a) (b)

Figure 3. Global downsampling of an irregular mesh. (a) Independent vertices are selected to be removed (empty circles). (b) Result of the removal of the selected vertices using half-edge collapse operations.

his multi-level smoothing algorithm. The global downsampling method selects an independent set of vertices, labeled *odd vertices*, to be removed by a sequence of half-edge collapses.[6, 16] The remaining vertices are labelled *even vertices*. The inverse operation (*i.e.* global upsampling) required for the recontruction operation re-inserts the previously removed vertices to create a mesh topologically identical to the initial one. The odd vertices are removed using half-edge collapse operations. The remaining vertices compose the coarse mesh at the output of the decomposition scheme. It is important to label the vertices and store those labels since they are used during the reconstruction. We will see that during the reconstruction we apply the relaxation operator first to the odd vertices and then to the even vertices to insure a smooth surface representation. Figure 3 illustrates one step of the downsampling algorithm. On the left part, independent vertices are selected to be removed (empty circles). The right part shows the result after the removal of the selected vertices using half-edge collapse operations.

3.2. Reconstruction

The reconstruction scheme is shown in Figure 4, and reconstruction is done in three steps:

1. Mesh upsampling
2. Odd vertex reconstruction
3. Even vertex reconstruction

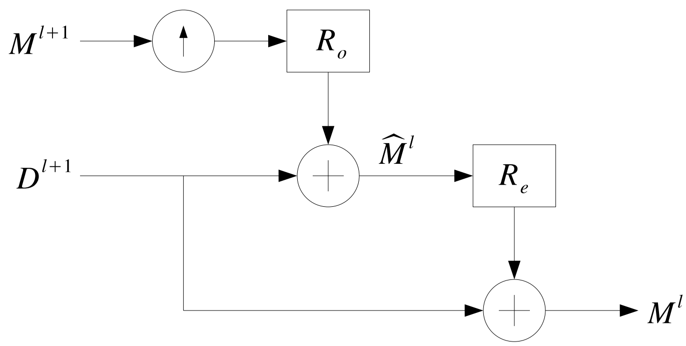


Figure 4. Reconstruction scheme of the multiresolution analysis framework for irregular meshes with attributes.

Starting from a coarse mesh M^{l-1}, the odd vertices are re-inserted using the *upsampling* operator (a series of vertex split operations). The position and attributes of the odd vertices are predicted using the *relaxation* operator R_o, and then exactly reconstruted by adding the corresponding detail coefficients D^{l-1} to create an intermediate mesh $\widehat{M}^l = (\widehat{V}^l, K^l)$:

$$\hat{\mathbf{a}}_{i,r}^{l} = \mathbf{R}(\mathbf{a}_{i,r}^{l}) + \mathbf{d}_{i,r}^{l-1}, \quad \forall i \in O^l, \tag{5}$$

where O^l is the set of indices of the odd vertices. Note that the intermediate mesh $\widehat{M}^l$ has the same topology K^l as the fine mesh M^l.

Finally the even vertices of the resulting mesh are relaxed and reconstructed by adding the corresponding detail coefficients.

$$\mathbf{a}_{i,r}^{l} = \mathbf{R}(\hat{\mathbf{a}}_{i,r}^{l}) + \mathbf{d}_{i,r}^{l-1}, \quad \forall i \in E^l, \tag{6}$$

where E^l is the set of indices of the even vertices.

The relaxation of even vertices after the reconstruction of the odd vertices is essential to get a smooth representation of the surface. This leads to an approximating decomposition. It is possible to build an interpolating decomposition by skipping the even vertex recontruction step.

4. DENOISING BY SOFT THRESHOLDING

In this section we present an application of the proposed multiresolution framework to mesh attribute denoising, and extend the concept of soft-thresholding to 3D meshes. Wavelet shrinkage is a popular method used to denoise data. The idea is to transform the data into the wavelet basis, where the large coefficients are mainly the signal, and smaller ones represent the noise. By suitably modifying these coefficients, the noise can be removed from the data. It is important to understand the wavelet shrinkage methods as a denoising algorithm and not as a smoothing algorithm. Denoising attempts to remove whatever noise is present and to retain whatever signal is present regardless of the frequency content, while smoothing removes high frequencies and retains low frequencies.[17] The soft thresholding method[18] has proved its efficiency and its robustness for denoising applications. Given a threshold τ for monodimensional data d, the rule $\delta_\tau(d) = sgn(d)\max(0, |d| - \tau)$ defines soft thresholding, the operator δ_τ nulls all values of d for which $|d| \leq \tau$ and shrinks toward the origin by an amount of τ all values for which $|d| > \tau$.

It is possible to develop and apply filtering operators to irregular meshes by suitably modifying the detail coefficients. Figure 5 shows the new reconstruction scheme of our multiresolution mesh analysis framework allowing the filtering of the detail coefficients. A threshold τ is apply to the filter operator F in order to process the detail coefficients before the reconstruction. By suitably modifying this threshold, one can obtain several signal processing filters.[7, 19]

In the setting of multiresolution analysis of irregular meshes, geometric details $\mathbf{d}$ are represented by a vector in 3D. Therefore, the soft thresholding rule has to be adapted to 3D vectors, but the principle remains the same. The operator nulls all values of $\mathbf{d}$ for which $\|\mathbf{d}\| \leq \tau$ and shrinks the vector $\mathbf{d}$ toward the origin by an amount of τ if $\|\mathbf{d}\| > \tau$. Given threshold τ for data $\mathbf{d}$ in 3D space, the soft thresholding is defined by:

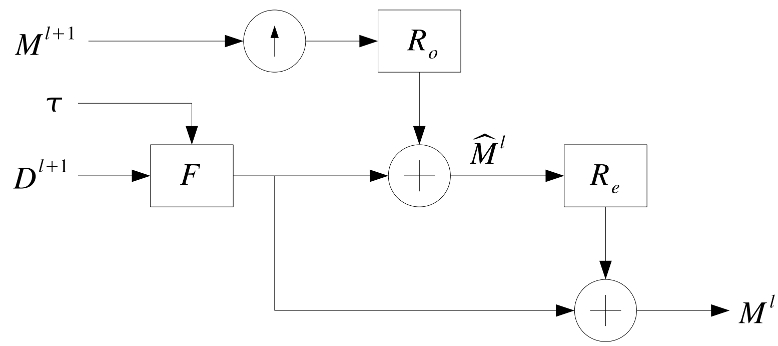


Figure 5. Filtering reconstruction scheme of the multiresolution analysis framework for irregular meshes with attributes.

$$\delta_\tau(\mathbf{d}) = \begin{cases} \mathbf{0}, & \|\mathbf{d}\| \le \tau \\ \mathbf{d} - \frac{\mathbf{d}}{\|\mathbf{d}\|}.\tau, & \|\mathbf{d}\| > \tau \end{cases}. \tag{7}$$

The soft-thresholding function determines how the threshold τ is applied to the detail coefficients. Other thresholding functions can be used such as hard-thresholding or garrote threhsolding. But the soft-thresolding function is known to be the most efficient.[20]

To complete the denoising method a shrinkage rule is needed to determine how the threshold τ is calculated. Several methods were proposed.[20] We choose the *BayesShrink*[21] rule to compute the threshold. It uses a Bayesian mathematical framework to derive data-dependent thresholds that are nearly optimal for soft thresholding. The formula for the threshold for a given vertex i on the level l is:

$$\tau_i = \frac{\hat{\sigma}_\varepsilon^2}{\hat{\sigma}_i^l}, \tag{8}$$

where $\hat{\sigma}_\varepsilon^2$ is the estimated noise variance, and $\hat{\sigma}_i^l$ is the estimated data variance at the vertex i on the level l. The noise variance $\hat{\sigma}_\varepsilon^2$ is estimated as the median deviation of the detail coefficients on the finest level. The estimate of the data variance for a vertex i is computed localy :

$$\hat{\sigma}_i^l = \max\left(0, \sigma_i^l - \hat{\sigma}_\varepsilon^2\right) \tag{9}$$

where

$$\sigma_i^l = \frac{1}{\#N_1^l(i) + 1} \sum_{j \in N_1^l(i) \cup i} \|\mathbf{d}_j^l\|^2 \tag{10}$$

with $\#N_1^l(i)$ the number of vertices in the 1-ring neighborhood $N_1^l(i)$ of the vertex i on the level l.

It is important to note that this is generic and can be applied on any attribute detail coefficients of a model. So this method is able to denoise any attributes attached to the model, assuming the attributes lie in a Euclidian space.

5. RESULTS

Figure 6 shows results of denoising the head model 6(a). This model was acquired with a Minolta VI-910 laser range scanner at the Université de Bourgogne. The noise comes from the limited accuracy of the scanner. This model was denoised using the soft thresholding on the four finest levels of details (Figure 6(c)) compared to the recent bilateral denoising method[15] (Figure 6(b)). Notice that the model is nicely smoothed in both case, while keeping important geometrical features such as sharp edges. The bilateral denoising can be seen as an anisotropic geometric diffusion process. It gives very good results, but we see some undulations on the surface of the model, which come from large scale noise. Trying to remove those undulations with the bilateral denoising will remove the important features of the mesh. Our method removes this noise due to the multi-scale properties of the framework. The soft thresholding is a fast, efficient denoising method and gives good results. Moreover, our method is a non-iterative method, and so it requires only one pass over the model.

(a) (b) (c)

Figure 6. Denoising of the head model in (a) using bilateral filtering (b) and soft thresholding (c).

Figure 7 shows the application of the soft-thresholding method to the colors of the model. The colors are defined on each vertex of the model, and these data are also corrupted by some noise. Fig. 7(a) shows the original model, and Fig. 7(b) shows the model after the geometric denoising using our method. Fig. 7(c) shows the results of the color denoising using soft thresholding. We see that the denoising is efficient and remove some speckles created by the acquisition noise.

6. CONCLUSION AND FUTURE WORK

In this paper, a framework for surface attribute denoising has been presented combining the multiresolution analysis and the soft thresholding methods.

Multiresolution analysis for irregular meshes is achieved by the decomposition and reconstruction algorithms of Guskov.[7] We use an improved subdivision method by using discrete-differential geometry operator reducing the computation to the 1-ring neighborhood while keeping the same efficiency.

Geometric and attribute noise on meshes was removed by applying the soft thresholding filter to the detail coefficients of the multiresolution analysis. This method was successfully employed on real world scene scanned models with different sources of noise. Our method is able to remove the measurement errors generated by 3D scanners while keeping the important features of the models (e.g. sharp edges, color transitions).

For future work, we will improve the denoising process by adding a feature detection algorithm in order to enhance the feature conservation such as the small details. We also plan to restructure the algorithm so that it can processed the three coordinates (x, y, z) independently, because usually 3D scanners do not have the same accuracy for each axis.

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Figure 7. Denoising the colors of the head model in (a). Only geometric denoising is shown in (b), and geoemtric and color denoising is shown in (c). (d)-(e) show a zoom of respectively the original model and the color denoised model.

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