On the use of multistability for image processing

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Abstract

We use the multistable property of a Cellular Nonlinear Network (CNN) to extract the regions of interest of an image representing the radiography of a soldering. We also propose the electronic implementation of the elementary cell of this CNN for a further electronic implementation of this network. We conclude with a discussion which enlarges the potential applications of this elementary cell in the field of nonlinear physics.

Key words: Cellular Nonlinear Network, Image processing, Nonlinear circuits.
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1 Introduction

Since the introduction of Cellular Neural/Nonlinear Networks (CNN) by L. Chua and L. Yang as a novel class of information processing systems [1,2], a lot of applications based on these nonlinear systems have been developed in a rich variety of fields like signal-image processing [3], non conventional method of computing [4], video coding [5,6], quality control by vision [7], cryptography [8] to cite but a few (see ref. [9] for an overview of the applications). This growing interest devoted to CNNs over the past ten years can be explained by their ability to solve problems of high computational complexity. Indeed, their massively parallel architecture combined with the genuine properties of nonlinear systems has allowed to develop powerful processing devices which can be realized with electronics circuits [10] or with active chemical media [11,12]. Among the complex applications which can be implemented using chemical photosensitive nonlinear media [11,12], one can cite finding the shortest path...
in a labyrinth [13], extracting image skeleton [14], restoring individual components of an image with overlapped components [15], or controlling mobile robot [16]. Even if some limitations of this chemical computers are now well known, it no way dismiss the efficiency and high prospects of processing inspired by nonlinear systems [17].

On the other hand, using analog nonlinear networks remains the most common way to implement CNNs because of the real-time processing capabilities of analog circuits [1]. Image segmentation [18], noise filtering with an equivalent Nagumo lattice [19], edge filtering using a two dimensional Nagumo network [20] contrast enhancement and gray level extraction with a nonlinear oscillators network [21–23] are few examples of electronic realizations of CNNs for processing applications. The aim of this letter is to present a new CNN for image processing purposes and to develop electronically the elementary cell of this new network. Furthermore, we propose to test our CNN with a real image, namely the noisy and weakly contrasted initial image of figure 1.\(\text{a}\) representing a digitalized radiography of a soldering between two rods of metal. Once the histogram has been re-scaled in the range \([0; 1]\), figure 1.\(\text{b}\) reveals four regions of interest

- The “background” in light gray which corresponds to the two rods of metal;
- A central part which represents the “soldered joint” in medium gray;
- Outside the “soldered joint”, a “projection” of metal occurring during the soldering of the two metals appears as a white spot;
- A dark gray spot which represents a “gaseous” inclusion inside “the soldered joint”.

In this paper, we investigate how multistability can be used in a diffusive CNN to extract the four regions of interest of this image. In the second section, we present the bistable CNN based on the standard Nagumo equation. Especially, owing to the bistable nature of this system, we show numerically that the pattern obtained do not allow the extraction of regions of interest. In the third section, changing the nonlinearity, we propose to increase the number of stable states to allow a better extraction of the defects. Then, in the last section, we electronically realize the elementary cell of the previous multistable CNN and we compare its behavior to the theoretical one. We finally conclude in the last section with a brief discussion of the possible applications of this CNN.

\section{The bistable CNN.}

We consider a \(\text{CNN}\) whose cell state \(X_{i,j}\), representing the gray level of the pixel number \(i, j\), obeys to the following set of equations:
\[
\frac{dX_{i,j}}{dt} = f(X_{i,j}) + D \sum_{(k,l) \in N_r} (X_{k,l} - X_{i,j}), \quad i, j = 1...N, 1...M, \tag{1}
\]

where \( f(x) \) represents the nonlinearity, \( N_r = \{i-1, i, i+1\} \times \{j-1, j, j+1\} \) is the neighborhood, \( N \times M \) the image size and \( D \) is a coupling constant. Moreover, the initial condition applied to the cell \( i, j \) of the network corresponds to the initial gray level \( X^0_{i,j} \) of the image to process. The image after a processing time \( t \) is obtained noting the state \( X_{i,j}(t) \) of all cells of the network at this specific time \( t \).

In the case of the Nagumo equation, \( f(x) \) is chosen cubic with three roots \( 0, a, 1 \) such that \( f(x) = -x(x-a)(x-1) \). The roots 0 and 1 correspond to the stable steady states of the system, whereas the nonlinearity threshold \( a \) defines the unstable state.

In image processing context, this CNN has demonstrated the possibility to perform edge extraction for \( a \neq 0 \) [20] and noise filtering [24] when \( a = 0.5 \). However, these processing tools both depend on the processing time, that is the time we let evolve the initial condition in eq. (1), which is quite subjective and hinder an automatic implementation of the processing task.

In this paper, we restrict our study to the case \( a = 0.5 \) and we focus our attention on the stationary case, that is when all cells of the network do not evolve any more. From a mechanical point of view and as represented in figure 2, eq. (1) also describes the evolution of an overdamped network of \( N \times M \) particles coupled with springs and submitted to a nonlinear force \( f(x) \) deriving from the double well potential \( \Phi(x) = - \int_0^x f(u)du \) (figure 2.(b)).

Therefore, according to the value of the nonlinear force \( f(X_{i,j}) \) compared to the elastic force \( D \sum_{(k,l) \in N_r} (X_{k,l} - X_{i,j}) \), the particle with initial position \( X^0_{i,j} \) is attracted in one of the two wells [25]. From an image processing point of view, it means that each pixel, owing to its initial gray level \( X^0_{i,j} \), will take a final gray level value close to the two potential minima, namely 0 (black) and 1 (white).

Therefore, for sufficiently large time, using a nonlinearity with two stable states, namely 0 and 1, provides only two regions of interest, which corresponds to an almost black and white pattern.

The bistable behavior of the CNN is revealed in figure 3 where we have numerically investigated the evolution of the filtered image versus the processing time \( t \). At the beginning, the noise is first removed for processing times \( t = 3 \) and \( t = 5 \) (figure 3.a and b respectively). However, due to the bistable nature of the CNN, the coherent structure of the image is destroyed since “the soldered joint” begins to disappear for \( t = 3 \) (figure 3.(b)). Moreover, for a greater processing time, namely \( t = 10 \), “the projection” is merged into “the background” in white (figure 3.(c)). Lastly, when all cells of the bistable CNN do not evolve any more, the image reaches the pattern represented figure 3.(d)
where “the soldered joint” has almost disappeared in the white “background”, as “the projection” did at the time $t = 5$ (figure 3(b)). The bistable CNN is thus inappropriate to realize extractions of regions of interest since it is not possible to extract all defects of the soldering.

3 The Multistable CNN

In this section, we propose to increase the number of stable states in order to obtain a pattern which conserves the coherent structure of the initial image and allow to extract all the regions of interest. Therefore, we are lead to consider a multistable CNN defined by eq. (1), but with the following nonlinear force:

$$f(x) = -\beta(n - 1) \sin \left[ 2\pi(n - 1)x \right].$$

(2)

deriving from the multiple wells potential $\Phi(x) = -\int_0^x f(u)du$ represented in figure 4. In eq. (2), $n$ provides the number of stable states, that is the number of wells, whereas $\beta$ adjusts the potential barrier height between two consecutive potential maxima.

In order to show the multistability of the systems let us consider first the uncoupled case, that is $D = 0$ in eq. (1).

The behavior of a cells is then ruled by:

$$\frac{dX_{i,j}}{dt} = -\beta(n - 1) \sin \left[ 2\pi(n - 1)X_{i,j} \right].$$

(3)

Solving eq. (3), the temporal evolution of a cell excited with an initial condition $X_{i,j}^0$ can be expressed under the following form

$$X_{i,j}(t) = \frac{1}{\pi(n - 1)} \left[ \arctan \left( \tan(\pi(n - 1)X_{i,j}^0)e^{-\beta(n-1)^2\pi t} \right) \right] + \frac{k}{n - 1},$$

(4)

where $k$ is the nearest integer of $(n - 1)X_{i,j}^0$.

The evolution of a cell for different initial conditions $X_{i,j}^0$ in the range $[0; 1]$ deduced with eq. (4) is confirmed in figure 5 by the numerical predictions. This simulation were performed by solving eq. (3) using a fourth order Runge-Kutta algorithm with an integrating time step $dt = 0.001$. After a transient, for a set of initial conditions below a threshold $V_{th1} = 1/8$ which corresponds to the location of the first potential barrier (dotted lines of figure 5), the cell evolves towards its first stable state 0. The same behavior is revealed for initial conditions in the range $[V_{th1}; V_{th2}] = [1/8; 3/8]$, except that the final state
of the cell corresponds exactly to the location of the second minima of the potential, namely $1/4$. The same remark can be done for all other ranges of initial conditions, which validate the multistable nature of the system.

We now numerically investigate the coupled case. As for the bistable CNN, we have simulated the multistable CNN described by eqs. (1) and (2) using the image of figure 1 as initial condition in the network. Owing to the multistable nature of the CNN, a pixel with initial gray level $X_{i,j}^0$ can now take a final gray level defined by one of the 5 possible stable states. The behavior of the multistable CNN is then summarized in figure 6 for three different processing times.

Unlike the bistable CNN, the noise is quickly removed and in the same time, the coherent structure of the image consisting of “the projection”, “the gas”, “the background” and “the soldered joint” is conserved (figure 6.(a) for $t = 0.1$ and (b) for $t = 0.5$). Lastly, for a sufficiently large time, the image no longer evolves and each defect appears with a different mean gray level corresponding to one of the potential minima. Standard threshold filtering allows then to extract all defects of the initial image, namely, “the background”, “the soldered joint”, “the gas” and “the projection”.

### 4 Sketch of the multistable CNN

This section is devoted to a discussion on the realization of the nonlinear network ruled by eqs. (1) and (2). Especially, we focus our attention on the electronic realization of the elementary cell of the multistable CNN. We also propose to validate its multistable behavior since this fundamental property allows the extraction of interest regions in image processing context.

#### 4.1 Equation of the nonlinear network

The CNN is realized by coupling the elementary cell of figure 7 to its eight neighbors with linear resistors $R$. The elementary cell of the CNN consists of a capacitor in parallel with a nonlinear resistor whose current-voltage characteristic can be approximated by the following sinusoidal law on the range $[-2V; 2V]$:

$$I_{NL}(U) \simeq IM \sin(2\pi U).$$  \hspace{1cm} (5)

This resistor is developed according to the methodology described in [26] and recalled in figure 8. A polynomial source realized with analog multipliers and operational amplifiers provides the voltage $P(U) + U$, where $P(U)$ is a poly-
nominal law. A resistor $R_0$ assumes a feedback between the input/output of the polynomial source, which provides the nonlinear current:

$$I_{NL}(U) = -P(U)/R_0.$$  \hspace{1cm} (6)

In order to obtain a sinusoidal law in the range $[-2V, 2V]$ with 9 zeros, we fit the sinusoidal expression (5) by a polynomial law in the range $[-2V, 2V]$ using a least squares method at the order 15. It provides then the coefficients of the polynomial source $P(U)$ to realize the sinusoidal current.

As shown in figure 9, the theoretical sinusoidal law of the current (solid line) is in good agreement with the experimental data (crosses). Note that, the weak discrepancies are mainly imputable to the polynomial approximation of the sinusoidal law by the least squares. Indeed, this slight discrepancy can be reduced increasing the order of the approximation. However, it increases the number of components used to realize the nonlinear resistor, which is not of crucial interest provided that the behavior of the elementary cell exhibit the multistable property.

Applying Kirchhoff laws to the node $i,j$ of the CNN, we obtain straightforwardly the equation of the network:

$$C \frac{dU_{i,j}}{d\tau} = -I_{NL}(U_{i,j}) + \frac{1}{R_0} \sum_{(k,l) \in N_r} (U_{k,l} - U_{i,j}),$$  \hspace{1cm} (7)

where $N_r = \{i - 1, i, i + 1\} \times \{j - 1, j, j + 1\}$ represents the neighborhood and $\tau$ corresponds to the experimental time.

Setting $\tau = tR_0C$, $\beta = \frac{I_{M}R_0}{(n - 1)^2}$, $U_{i,j} = X_{i,j}(n - 1) - 2$, and $D = \frac{R_0}{R}$,

the equation of the network reduces to

$$\frac{dX_{i,j}}{dt} = \frac{P(X_{i,j}(n - 1) - 2)}{n - 1} + D \sum_{(k,l) \in N_r} (X_{k,l} - X_{i,j}).$$  \hspace{1cm} (8)

An analog simulation of the normalized CNN obeying to eqs. (1) and (2) is then realized, since for $X_{i,j} \in [0; 1]$, corresponding to $U_{i,j} \in [-2V; 2V]$

$$P\left[ X_{i,j}(n - 1) - 2 \right] = -I_{NL}\left[ X_{i,j}(n - 1) - 2 \right]R_0$$

$$\simeq -\beta(n - 1)^2 \sin(2\pi(n - 1)X_{i,j}).$$  \hspace{1cm} (9)
4.2 Behavior of the elementary cell of the CNN

Considering the uncoupled case (that is \( R \rightarrow \infty \)), we now investigate experimentally the response of the elementary cell to different initial conditions \( U^0_{i,j} \) in the range \([-2V; 2V]\). The behavior of the cell is summarized in figure 10 where we have reported the experimental potential obtained integrating the nonlinear current. As for the theoretical analysis presented in section 3, this figure clearly reveals the multistable behavior of the elementary cell. Indeed, there exists 5 ranges of initial conditions defined by the position of the potential barriers which determine the final state of the cell. Moreover this final state is reached after a transient and corresponds exactly to the position of one of the 5 potential minima.

Note that the worths of the feedback resistor \( R_0 \) and of the capacitor \( C \) adjust the length of the transient and can be tuned to match real-time constraint.

5 Conclusion

We have extracted the regions of interest of an image by using the multistable nature of a CNN ruled by reaction-diffusion equations. Unlike some existing CNNs [15], this image processing task is performed without tuning the processing time since the filtered image is deduced when all cells of the CNN do not evolve any more. This property could be of crucial interest for a further implementation of this CNN in a hardware device. Moreover the nonlinear resistor presented in this paper could be useful to design an experimental electrical lattice modelling the Sine-Gordon equation [27]. Experiments on supratransmission phenomenon or breather generation [28,29] could then be quantitatively realized. Therefore, this work constitute a framework for further studies and experiments in nonlinear science and its applications to signal-image processing. In particular, noise enhancement of subthreshold details via stochastic resonance phenomenon [30,31] could be investigated in our multistable device.

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References


Figure 1: (a): Weak contrasted image to process provided by laboratory LCND CEA Valduc, France. (b): Enhanced image in the range [0; 1] with its corresponding histogram, the arrows indicate the regions of interest which must be extracted.

Figure 2: Mechanical point of view of the CNN. The cell with coordinates \(i, j\) is analog to an overdamped particle which is coupled to its 8 neighbors by springs with strength \(D\) (see (a)). This particle is also submitted to a nonlinear force \(f(X_{i,j}) = -X_{i,j}(X_{i,j} - 0.5)(X_{i,j} - 1)\) depending on the particle displacement \(X_{i,j}\). According to the resulting force applied by the springs of strength \(D\), the particle with initial position \(X_{i,j}^0\) is attracted in one of the two wells of the onsite potential represented in (b) from which derives the nonlinear force.

Figure 3. Images obtained with the bistable CNN. Parameters: \(a = 0.5, D = 0.025\). Processing time: (a) \(t = 3\), (b) \(t = 5\), (c) \(t = 10\), (d) \(t = 3000\).

Figure 4. Multistable potential represented for \(n = 5\) and \(\beta = 0.25\). A pixel with initial gray level \(X_{i,j}^0\) is analog to a particle submitted to a resulting elastic force which may induces possible transitions between the five wells of the potential.

Figure 5. Evolution of a cell in the uncoupled case for different initial conditions. The theoretical expression (4) is represented in solid line whereas the (o) signs are obtained by solving numerically eq. (3). At the right the potential provides a reference.

Figure 6. Image obtained with the multistable CNN. Parameters: \(n = 5, \beta = 0.25, D = 1.4\). (a) \(t = 0.1\), (b) \(t = 0.5\), (c) \(t = 50000\).

Figure 7. Sketch of the CNN. In a sake of clarity, only the elementary cell of the node \((i, j)\) of the CNN is represented at the right.

Figure 8. The nonlinear resistor (left) and its equivalent sketch using a polynomial source circuit (right). \(R_0 = 2K\Omega\).

Figure 9. Current-voltage characteristic of the nonlinear resistor (crosses) compared to the sinusoidal law (solid line). The parameters are \(R_0 = 2K\Omega, C = 390nF, I_M = 2mA\).

Figure 10. Response of the elementary cell in the uncoupled case to different initial conditions \(U_{i,j}^0, R_0 = 2K\Omega, C = 390nF, I_M = 2mA\). The experimental potential obtained integrating the nonlinear current (crosses) provides a
reference. The theoretical current and potential are superimposed on solid line.
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Fig. 3. S. Morfu, B. Nofiélé and P. Marquié.
Fig. 4. S. Morfu, B. Nofielé and P. Marquié.
Fig. 5. S. Morfu, P. Marquié and S. Morfu.
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Fig. 7. S. Morfu, B. Nofielé and P. Marquié.
Fig. 8. S. Morfu, B. Nosiélé and P. Marquié.
Fig. 10. S. Morfu, B. Nofiélé and P. Marquié.

\[
\Phi(U) = \int_0^U I_{NL}(V) dV
\]