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EXPERIMENTS ON AN ELECTRICAL NONLINEAR OSCILLATORS NETWORK

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We have recently proposed a Cellular Nonlinear Network (CNN) based on nonlinear oscillator properties to perform image processing tasks. We present here the electronic implementation of the elementary cell of this CNN. We experimentally verify the main property of the CNN, that is the possibility to enhance a weak difference of initial condition between two specific cells of the CNN at a given time. For this optimal time, a contrast enhancement of a weak contrasted gray scale is possible.

Keywords: nonlinear signal processing; nonlinear circuits.

1. Introduction

Since the pioneer work of L. Chua concerning Cellular Nonlinear Networks (CNN) [Chua (1998)], it has become clear that nonlinear media can be regarded as parallel multiprocessors systems dedicated to signal processing. In these nonlinear systems, the signal amplitude constitutes an additional dimension which offers a rich variety of properties not shared by linear systems. Image processing with nonlinear networks [Adamatzky *et al*. (2002);

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Rambidi *et al*. (2002); Perona and Malik (1990); Julián *et al*. (2002); Morfu and Comte (2004)], signal detection/transmission via the nonlinear stochastic resonance phenomenon [Gammaitoni *et al*. (1998); Godivier *et al*. (1999); Morfu *et al*. (2003)] are few examples of a nonrestrictive list where taking into account nonlinearity allows to transcend the limitation of classical linear processes.

Note that until now, most of the electronically implemented CNN are based on reaction-diffusion systems and perform different tasks such as edge extraction in image processing field [Comte *et al*. (2001)], or noise filtering in signal processing area [Marquié *et al*. (1998)]. However, it is also possible to use the properties of nonlinear inertial systems to develop image processing tools. Indeed, recently, we have theoretically and numerically introduced a CNN based on the properties of nonlinear oscillators to perform a contrast enhancement of a weak contrasted image [Morfu and Comte (2004)]. This paper is mainly devoted to the electronic implementation of such a CNN. We first present the electrical lattice made of uncoupled nonlinear oscillators. Then, we show theoretically and experimentally that two oscillators with a slight difference of initial amplitude can present a maximum amplitude difference at a given time. This main property is experimentally confirmed and allows, as widely discussed in [Morfu and Comte (2004)], to perform a contrast enhancement of an image.

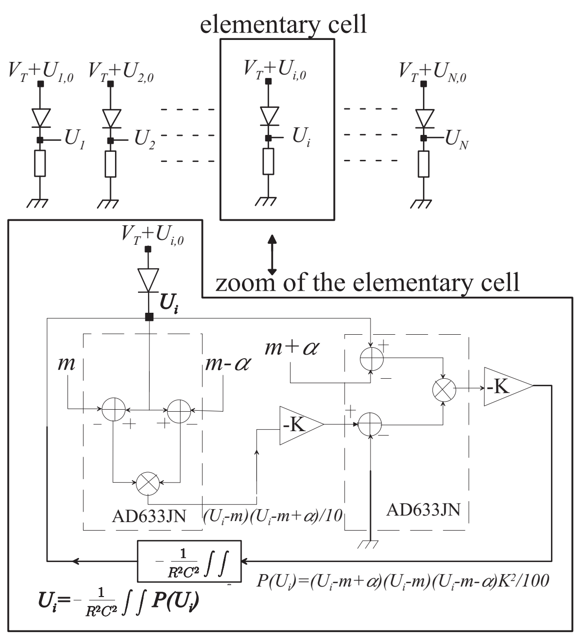

Fig. 1. Sketch of the 1D CNN and its elementary cell.

2. The nonlinear lattice

We consider a lattice of N elementary cells consisting of a nonlinear circuit and a $1N4148$ diode as represented in figure 1. Each nonlinear circuit includes a polynomial source $P(U) = K(U-m+\alpha)(U-m)(U-m-\alpha)/100$ made with AD633JN analog multipliers and standard amplifier circuit with amplification $-K$ to balance the scale factor $1/10\ V^{-1}$ of the multipliers. The worth of the zeros $m+\alpha$, $m-\alpha$ and m is adjusted, for all cells, with only three external voltage sources to ensure a strict homogeneity in the lattice. Moreover, a double integrator realized with classical operational amplifier circuit, resistor R and capacitor C assumes a feedback between the input/output of the polynomial source (figure 1). According to the sketch of figure 1 and setting $K' = K/10RC$, the voltage U_i at the diode cathode of the i^{th} cell obeys to:

$$\frac{d^2U_i}{dt^2} = -K'^2(U_i-m+\alpha)(U_i-m-\alpha)(U_i-m). \tag{1}$$

Furthermore, the initial condition $U_{i,0}$ of the i^{th} cell is introduced at the diode anode, by adding an offset voltage equal to the diode threshold $V_T = 0.7V$. The distribution of initial condition is considered to be linear and growing versus the cell number i such that $U_{i,0} = (i-1)\times 0.5/(N-1)$. Note that in image processing context, this distribution of initial conditions could be considered as a discrete gray scale.
Setting $x_i = U_i - m$, eq. (1) can be normalized as

$$\frac{d^2x_i}{dt^2} = -K'^2 x_i(x_i-\alpha)(x_i+\alpha). \tag{2}$$

Solutions of eq.(2) for a zero initial velocity are given by the following Jacobian elliptic functions [Abramowitz and Stegun (1970)]:

$$x_i(t) = x_{i,0}\ cn(\omega_i t, k_i), \tag{3}$$

where $x_{i,0}$, ω_i and $0 \le k_i \le 1$, correspond respectively to the oscillations amplitude, the pulsation and the modulus of the Jacobian elliptic function cn.
Using the properties of the cn function and deriving twice equation (3), we get

$$\frac{d^2x_i}{dt^2} = -\frac{2\ k_i\ \omega_i^2}{x_{i,0}^2}\ x_i\ [x_i^2 - \frac{2\ k_i - 1}{2k_i} x_{i,0}^2], \tag{4}$$

which provides, after identification with eq.(2), the pulsation ω_i and modulus k_i of the cn function

$$\omega_i(x_{i,0}) = K' \sqrt{x_{i,0}^2 - \alpha^2} \qquad \text{and} \qquad k_i(x_{i,0}) = x_{i,0}^2/2(x_{i,0}^2 - \alpha^2). \tag{5}$$

Writing the initial condition $U_{i,0} = x_{i,0} + m$, solutions of eq. (1) can be straightforwardly deduced from eqs. (3) and (5), namely

$$U_i(t) = m + (U_{i,0} - m) cn(\omega_i t, k_i), \tag{6}$$

$$\text{with} \quad \omega_i(U_{i,0}) = K'\sqrt{(U_{i,0}-m)^2 - \alpha^2} \quad \text{and} \quad k_i(U_{i,0}) = \frac{1}{2}\frac{(U_{i,0}-m)^2}{(U_{i,0}-m)^2-\alpha^2} \tag{7}$$

Both parameters ω_i and k_i of the i^{th} cell appear then as driven by the initial condition $U_{i,0}$ applied to that cell.

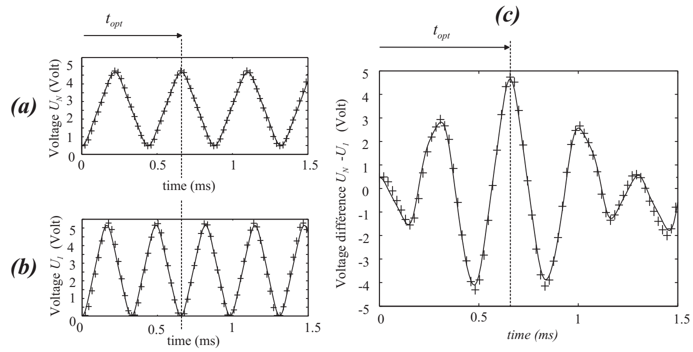

Fig. 2. Time evolution of the cells 1 (a) and N (b) and of their oscillations difference (c). Parameters: $\alpha = 1.02\ V$, $m = 2.58\ V$, $K = 10$, $U_{1,0} = 0\ V$, $U_{N,0} = 0.5\ V$, $R = 10\ K\Omega$, $C = 10\ nF$. $(+)$: Experimental results, solid line: Theoretical results obtained with Eqs. (6-7). The time to reach a phase opposition is $t_{opt} = 0.66\ ms$.

3. Behavior of the nonlinear electrical chain

To illustrate the behavior of the chain, we have reported fig. 2 the oscillations of the cells corresponding to the two ends of the lattice, namely cells $i = 1$ and $i = N$ with respective initial conditions $U_{i,0} = 0\ V$ and $U_{N,0} = 0.5\ V$. The oscillations take place in the range $[0\ V;\ 5.16\ V]$ for cell 1 and in $[0.5\ V;\ 4.66\ V]$ for the cell N as predicted by the range $[U_{i,0};\ 2m - U_{i,0}]$ for $i \in \{1, N\}$ deduced from eq. (6). Moreover, the two oscillators quickly achieve a phase opposition when $U_N = 4.66\ V$ and $U_1 = 0\ V$, involving a maximum oscillation difference $U_N - U_1 = 4.66\ V$ at the time $t_{opt} = 0.66\ ms$ (dotted line in figure 2). The weak initial amplitude difference $\varepsilon = U_{N,0} - U_{1,0} = 0.5\ V$ is then strongly increased at the time $t_{opt} = 0.66\ ms$.

In order to obtain the dynamics of the whole chain, we have plotted the voltage $U_i(t_{opt})$ reached by each cell at the time t_{opt} versus its initial condition $U_{i,0}$ (figure 3). Due to the nonlinearity $P(U)$ of the system, this curve is not linear, which means that all initial amplitudes are not increased by the same scale factor.

This property can be extended to the field of image processing when a 2-dimensional network is considered [Morfu and Comte (2004)]. Indeed the initial condition $U_{i,0}$ would correspond to the gray level of a pixel of a weak contrasted image, and $U_i(t_{opt})$ would be its gray level after a processing time t_{opt}. Therefore, in image processing context, the horizontal axis of fig. 3 would represent the initial gray scale of the weak contrasted image while the vertical axis would correspond to the gray scale of the processed image (the dynamics of both gray scales being defined by $0\ V$ for black and $4.66\ V$ for white).

Such a 2-dimensional system is described by the set of $N \times M$ differential equations:

$$\frac{d^2 V_{i,j}}{dt^2} = -K'^2 (V_{i,j} - m + \alpha)(V_{i,j} - m - \alpha)(V_{i,j} - m), \tag{8}$$

where $N \times M$ is the image size, $i = 1, 2 \ldots N$, $j = 1, 2 \ldots M$ and $V_{i,j}$ represent respectively the pixel coordinates and its corresponding gray level. Under these conditions, the behavior

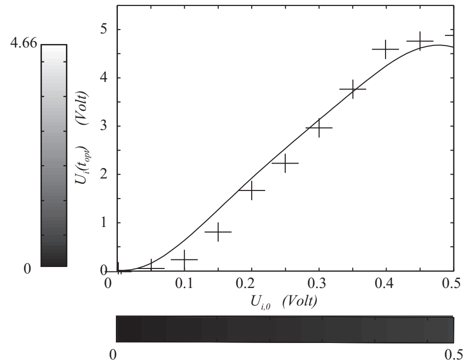

Fig. 3. State of the lattice at the optimal time $t_{opt} = 0.66$ ms versus its initial state. Parameters: $m = 2.58$ V, $\alpha = 1.02$ V, $K = 10$, $R = 10$ $K\Omega$, $C = 10$ nF. $(+)$: Experimental results; solid line: Theoretical results obtained with Eqs (6-7).

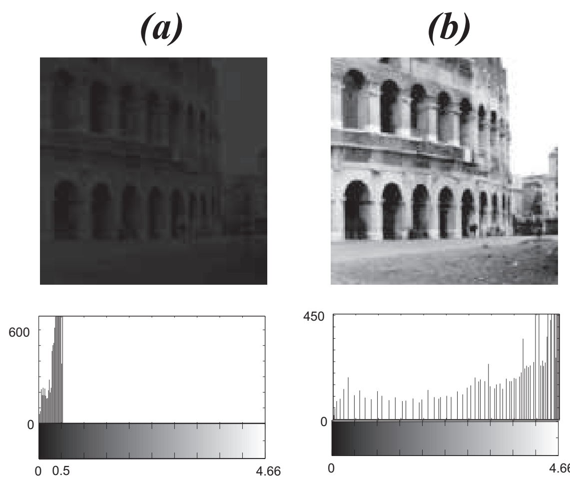

Fig. 4. a: Weak contrasted picture of the famous Coliseum and its histogram. b: Enhanced image and its histogram obtained with a direct simulation of Eq. (8). Parameters: $\alpha = 1.02V$, $m = 2.58$ V, $K = 10$, $N = M = 128$, time of simulation $t_{opt} = 0.66 \times 10^{-3}$ s, $R = 10$ $K\Omega$, $C = 10$ nF.

of the 2D electronic CNN can be predicted loading the weak contrasted image of figure 4.a as initial condition and simulating eq. (8) with a fourth order Runge-Kutta algorithm. Moreover, the range of initial conditions is still $[0,\ 0.5]$ as shown by the histogram of the weak contrasted picture of figure 4.a.

After a simulation time $t_{opt} = 0.66 \times 10^{-3}$, the range of gray levels of the resulting image is strongly increased and becomes $[0,\ 4.66]$ as for the 1D-network (histogram of figure 4.b). Therefore, for the specific processing time $t_{opt} = 0.66 \times 10^{-3}$, a contrast enhancement allows to reveal the famous Coliseum (figure 4.b) hidden in figure 4.a. As for the 1D-network, it is important to note that the nonlinear processing, performed by the system described by eq. (8), does not correspond to a multiplication of all gray levels by the same scale factor.

4. Conclusion:

In this paper, we have electronically realized an elementary cell of a *CNN* which was previously introduced for image processing purpose in ref. [Morfu and Comte (2004)]. We have shown experimentally that, in a 1D network, two elementary cells with a weak difference of initial conditions can present a maximum amplitude difference at an optimal time. For this optimal time, the response of the whole network is in good agreement with the theoretical prediction, which allows to validate the experimental device. Moreover, a contrast enhancement of an image is possible for this specific processing time as reported in [Morfu and Comte (2004)]. Therefore, this experimental network constitutes a framework for further studies and applications of nonlinear science in signal and image processing areas. The influence of linear and/or nonlinear coupling between cells is actually under investigation and may lead to new interesting properties of this CNN.

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