**SUSAN 3D OPERATOR, PRINCIPAL SALIENCY DEGREES AND DIRECTIONS EXTRACTION AND A BRIEF STUDY ON THE ROBUSTNESS TO NOISE**

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**ABSTRACT**

In order to complete the work on the 3D SUSAN operator, an extraction of principal saliency degrees and direction is proposed. This will lead the proposed operator to a complete classification of vertices in five different classes: cavity points, valley points, flat points, prong points and salient points. To assess the operator, we propose two different studies. The robustness to the resolution and the regularity of meshes is presented for three different operators. To complete the evaluation, the robustness to noise is analysed, for the Stokely and the SUSAN operator. Our conclusion presents the benefits, drawbacks of the technique and future works.

**Index Terms**— curvature, salient point detection, saliency, point characterization, SUSAN, robustness to noise.

1. **INTRODUCTION**

Analysis of 3D meshes is one of the most important research topic due to the recent diffusion, in the industry, of 3D scanning on production lines and increasing needs of precise measures. Most recent techniques, based on curvature analysis, cannot compute the curvature in the presence of a geometric discontinuity. Researchers [1, 2] assume that the mesh is sufficiently smooth for their analyses. In addition, the presence of a sharp edge infers additional computations like isotropic remeshing [2] that is not essential when inspecting industrial products for example. Motivated by this problem, the method presented in this article has been developed in order to directly give a reliable measure of vertices whatever the quality of the acquisition. Our goal is to define a 3D operator that classifies the five different types of vertices without any curvature computation. The extracted characteristics, such as curvature, can be used for mesh simplification, segmentation or denoising. A brief recall of the 2D SUSAN operator and its 3D extension are presented. A comparison method between the curvature measures and the saliency degree measures is described. In addition, two studies are summarized: the first study is focused on the robustness of the measure to the regularity and the resolution of meshes. The second deals with the robustness of the measures to noise. We conclude with benefits, drawbacks of the technique and orientation of future works.

2. **PRINCIPLE OF THE OPERATOR**

2.1. **2D SUSAN operator**

SUSAN (Smallest Unvalue Segment Assimilating Nucleus) method [3] is based on a circular window in which the center pixel, named nucleus, is the analyzed pixel. A pixel attribute is then chosen, for example the grey level in 2D images. In the circular window, the USAN area is defined as the set of pixels presenting a close value (bounded by a threshold) to the nucleus attribute. The operator response is the ratio of the USAN area over the total area of the circular window. In this way, a salient point corresponds to a minimum response of the SUSAN operator. We named the response of the SUSAN operator the saliency degree.

2.2. **3D SUSAN operator**

Our goal is to extend the 2D SUSAN operator to 3D meshes. The homologous 3D operator to a circular pixellized window becomes a voxellized sphere. By centering the voxellized sphere at a vertex, the mean saliency degree computation, due to the isotropy of the circular analysis window, depends on the volume of the intersection of a ball and the surface neighborhood of the inspected point. At this point, the mean saliency degree is computed by determining the USAN volume \( V_{USAN} \) that represent this intersection. The mean saliency degree is defined by:

\[
S_d = \frac{V_{USAN}}{V_{Sph,Vox}}.
\]

As proven in [4], the saliency degree is related to the curvature for sufficiently smoothed surfaces. Indeed, the saliency degree can be computed in every directions \( e_\theta \) perpendicular to the normal. For a 3D model, the mean saliency degree can also be expressed by:

\[
S_d = \frac{2\pi}{0} S^N_d(\theta)d\theta,
\]

where the normal saliency degree could be defined as \( S^N_d(\theta) = S^1_d\cos^2(\theta) + S^2_d\sin^2(\theta) \), with \( S^1_d \) and \( S^2_d \) the principal saliency degrees of the surface, and \( e_1 \) and \( e_2 \) the orthogonal principal directions of saliency. On a sufficiently smooth
surface, the SFM (Surface Fitting Methods) techniques extract the normal curvature by integration:

$$\kappa = \frac{2\pi}{\int_{0}^{\pi} \kappa^N(\theta) d\theta},$$  \hspace{1cm} (3)$$

with $\kappa^N(\theta) = \kappa_1 \cos^2(\theta) + \kappa_2 \sin^2(\theta)$ for every unit direction $e_\theta$ in the tangent plane. For every direction, [4] presents the following relation between normal saliency degree and normal curvature:

$$S_d = \frac{A_{USAN}}{\pi R^2} = \frac{1}{2} + \frac{\kappa(d^2 - d^2_0)}{6\pi R^2} + \frac{\alpha}{\pi} - \frac{\sin(2\alpha)}{2\pi}.$$  \hspace{1cm} (4)$$

3. PRINCIPAL SALIENCY DEGREE VALUES AND DIRECTIONS

By analogy with curvature operators, to achieve a complete characterization of vertices, the mean saliency degree measure described above must be completed by the measures of principal saliency degrees and directions. Only these informations can discriminate cavity and salient edges from valleys and ridges.

3.1. Approximation of the normal

We named the voxellized sphere $\epsilon$-ball, by analogy with the work of U.Clarentz et al. [5]. The normal at a vertex is approximated by subtracting the zero moment of the gravity center of faces that are included in the $\epsilon$-ball as:

$$N_{\epsilon-ball} = M_{CGF, \epsilon-ball} - C_0.$$  \hspace{1cm} (5)$$

Similarly, an approximation of the normal can be computed by subtracting the inspected vertex $C_0$ and the zero moment of all the gravity centers of the voxels included in the $V_{USAN}$ (intersection of the $\epsilon$-ball and the local patch of faces):

$$N_{\epsilon-ball} = C_0 - M_{CGV, \epsilon-ball}.$$  \hspace{1cm} (6)$$

3.2. Extraction of the principal saliency directions and degrees

The principal saliency directions are extracted by projecting the centres of voxels of the USAN volume onto the tangent plane and by computing the inertia moments of the projected points. Then, principal saliency degrees are computed in each principal direction as the ratio of the USAN area (intersection of an $\epsilon$-disk and the patch of faces around the inspected vertex) over the total area of the $\epsilon$-disk. With the principal saliency degrees and directions, our operator can classify five types of edges: cavity, valley, flat, prong and salient. Indeed, if the two principal saliency degrees extracted along the two principal saliency directions are similar, the projected points on the tangent plane fit a disk, and the information of the mean saliency degree leads to the type of edge (cavity, flat or salient). In the case of different principal saliency degrees, the projection fits an ellipse, and the edge is identified as a valley or a ridge depending on the mean saliency degree computed. To classify a vertex, we analyse the principal saliency degrees. If the principal saliency degrees are near $(s_2/s_1 > 0.8)$, the mean saliency value gives the type of the edge: cavity, flat or salient. If principal saliency degrees are different $(s_2/s_1 < 0.8)$, the mean saliency degree gives the type of the edge: valley or ridge. Figure 1 shows results of classification with principal saliency directions displayed only for valleys and ridges.
4. ACCURACY OF SALIENCY DEGREE MEASURES

4.1. How to compare different methods

The first (and naive) idea is to compare directly the results of the SUSAN 3D, the mean saliency degree \( S_d \), with the results of a mean curvature operator, the mean curvature \( \kappa \), but the comparison is not consistent, because the results of the operators must be compared in the same space parameter. A simple strategy to transpose the curvature results to the saliency space, consist in the use of principal curvatures extracted. Indeed, the mean curvature can be deduced from the principal curvatures by the formula:

\[
\kappa = \frac{\kappa_1 + \kappa_2}{2}.
\]

The advantage of using equation 7 is that the formula 1 can be applied on each principal curvatures, and the \( \overline{S_d} \) can be deduced from the results of the curvature operator. Once the two responses of the operators are in the same saliency degree space, they can be compared.

4.2. Strategy for an objective measure

To measure the accuracy of our operator, different tests have been carried out on objects with constant curvature: spheres. Three different spheres, obtained by subdivision, with the following resolutions have been used: 1280, 5120 and 20480 faces. For each sphere, a pseudo-regular and an irregular mesh have been created. The difference between regular and irregular meshes can be visualized in the Fig 2 and 3. Radii of the spheres have been fixed to three, in order to define a mean saliency degree reference. Formula 1 leads to the fixed reference, \( S_d = 0.475408 \). To assess the technique, two operators have been implemented: the Meyer’s operator [1] and the Stokely’s operator [6]. The first to compare the accuracy in the 1Ring (first face neighbors surrounding a vertex) and the second for a larger neighborhood. These two methods extract for each vertex the mean, Gaussian, and principal curvatures that are required, as explained in section 4.1., for an objective comparison to the mean saliency degree reference. In order to respect the width of the Meyer’s operator that is restricted to the 1Ring, the widths of our operator and the Stokely’s operator have been chosen in function of maximum distances of vertices of the 1Ring on the whole mesh. Tables 1 and 2 show the radii of the SUSAN sphere used for the Stokely’s and our operator.

4.3. Accuracy in the 1Ring neighborhood

From Table 1, we can observe a good behavior of our algorithm. Indeed, the more refine the mesh is, the better the saliency degree measures. In addition, we can notice that standard deviation errors with our operator are more stable and less dependant to the mesh regularity than other operators. The precision of the Meyer’s operator is better. Its behavior is inversed compared to other operators. Indeed, the subdivision algorithm increases the number of vertices, and the error introduced on each new vertex is added to the standard deviation error. Standard deviation errors of the Meyer’s operator increase as the number of vertices. We also verify that the regularity of the mesh is an important factor for the Meyer’s operator.

<table>
<thead>
<tr>
<th>Standard Deviation error</th>
<th>( R_{SUSAN} )</th>
<th>Meyer’s operator</th>
<th>Stokely’s operator</th>
<th>Our operator</th>
</tr>
</thead>
<tbody>
<tr>
<td>a</td>
<td>0.5</td>
<td>1.67e-6</td>
<td>0.0369</td>
<td>0.0262</td>
</tr>
<tr>
<td>b</td>
<td>0.25</td>
<td>3.18e-6</td>
<td>0.0184</td>
<td>0.013</td>
</tr>
<tr>
<td>c</td>
<td>0.125</td>
<td>6.45e-6</td>
<td>0.00922</td>
<td>0.00656</td>
</tr>
<tr>
<td>d</td>
<td>0.6</td>
<td>0.00517</td>
<td>0.045</td>
<td>0.0265</td>
</tr>
<tr>
<td>e</td>
<td>0.45</td>
<td>0.0077</td>
<td>0.0324</td>
<td>0.0156</td>
</tr>
<tr>
<td>f</td>
<td>0.35</td>
<td>0.00796</td>
<td>0.0252</td>
<td>0.0103</td>
</tr>
</tbody>
</table>

Table 1: Standard deviation errors of saliency degree, obtained on spheres with regular (a, b, c) and irregular (d, e, f) meshes: a) and d) 1280 faces, b) and e) 5120 faces, c) and f) 20480 faces.
Table 2: Standard deviation errors of saliency degree, obtained on spheres with regular (a, b, c) and irregular (d, e, f) meshes: a) and d) 1280 faces, b) and e) 5120 faces, c) and f) 20480 faces with $R_{SUSAN} = 0.5$.

Table 3: Standard deviation errors of saliency degree, obtained on the sphere 1280 regular faces with $R_{SUSAN} = 1$.

### 4.4. Accuracy in a larger neighborhood

Results in Table 2 show an accuracy of our operator about two times more precise compared to the Stokely’s operator. The same behavior of the two operators can be observed that is, the more refine the mesh is, the better the saliency degree measures. The two operators also demonstrate robustness to the irregularity of meshes in a larger neighborhood.

### 5. ROBUSTNESS TO NOISE

For this study, we have analyzed a real scanner by scanning a plane from 5 to 40 m by steps of 5 m with different resolutions: 2, 4, 6, 10 mm. By fitting a plane [7] and computing the standard deviation error at each vertex of the resulting points clouds, we found a mean standard deviation of 2.298 mm with a Gaussian form, which corresponds to the constructor specifications. To measure the robustness to noise, we used the sphere 1280 faces noised with six different Gaussian white noise models. Figures 4 and 5 show mean saliency degrees computed for two noised spheres. Table 3 summarizes standard deviation errors obtained on the different noised spheres. Results prove that our operator has a good resistance to noise compared to the results obtained with the Stokely’s operator. We can note a quick increasing of the standard deviation errors of our operator for the last three rows. This is due to the presence of spurious edges created by the noise. These spurious edges can be visualized in the Figure 5. Our operator is sensible to discontinuities but it is still more accurate compared to the Stokely’s operator.

### 6. CONCLUSION AND PERSPECTIVES

The proposed method gives reliable and useful characterization of vertices. Our operator can achieve a complete characterization of vertices in five classes with help of the extraction of principal saliency degrees and directions. The only parameter required for using the proposed operator is the radius of the SUSAN sphere that can be determined easily by a first preview of the object. The first test series have demonstrated the accuracy and the robustness to the regularity of meshes of our operator. Moreover, the tests have emphasized the good behavior of our operator along the different resolutions and his ability to correctly measure the local geometry of inspected objects in a near or extended neighborhood. The second test series have proved the robustness to noise and the reliability of our operator when applied on noised meshes. Future works are focused on the study of a multi-scale scheme and its use to denoising.

### 7. REFERENCES


