MIRROR-ADAPTED MATCHING OF CATADIOPTRIC IMAGES

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ABSTRACT

Images produced with catadioptric sensors contain a significant amount of radial distortions and variation in inherent scale. Direct application of conventional methods of image processing without considering the mirror anamorphosis yields erroneous issues. Examples of such applications are those based on the pixel neighborhood: one can mention image derivation, image convolution or image matching. The inefficiency of classical algorithms in catadioptric images processing leads in general to biased results. In this paper, we propose a new method to match a pair of catadioptric images taking into account their distorted topology. This method is based on an adapted window resulting from a convenient parameterization of the mirror. Experiments have been performed on image sequences acquired with a parabolic catadioptric sensor.

Index Terms— Catadioptric vision, image processing, matching, Harris detector, adapted neighborhood

1. INTRODUCTION

Catadioptric sensors are systems consisting of convex mirror and conventional camera (CCD or CMOS) [9]. The used mirrors can include several convexes shapes namely paraboloidal, spherical, hyperboloidal, conical, ellipsoidal or planar. They produce a hemi-spherical field of view [8]. Catadioptric devices are largely used in telescopes, teleconference, virtual reality, monitoring and recently in the navigation of mobile robots.

Image processing is mostly based on pixel neighborhood. For instance, the filtering of an image takes into account the pixel neighborhood information to carry out the necessary convolutions within the image. Another interesting application is the correspondence between a pair of images. It consists, first of all, to detect points of interest in the images, and then to compare the information carried by their respective neighborhood to match them. Therefore, if a pixel neighborhood is wrong detected, the processing will be biased and erroneous.

Our interest in this work is to perform the matching of a pair of catadioptric images [2]. The choice of a pixel neighborhood in this case is crucial. The anamorphosis of those images obliges a straight line to be a curve [6]. Two neighbor pixels do not lie on the same vertical or horizontal lines anymore, but belong to a curve having the same deformation as the mirror that composes the catadioptric sensor.

Several authors deal with this question. Daniilidis [3] uses a projective space where operators are shift variant. The catadioptric image is projected on a virtual sphere and the convolution of the image is carried out on that space. The final result is then obtained by a stereographic projection on the image plane. Strauss [10], [6] takes a cylinder as a projective plane in order to perform edge detection or to define morphologic operators on catadioptric images. All these methods are based on projective spaces with known parameters, but there is no direct method to process catadioptric images on their own support. In addition, the original image projection onto another space may result in the lost of relevant information. Therefore, an interpolation step is needed to approximate the result.

The proposed method does not need any projective space. It is based on an adapted window surrounding a pixel which respects perfectly the topology of catadioptric images. The shape of the mirror has to be primarily known to design the image mask. The neighborhood of a pixel is taken firstly on the mirror and then projected onto the image plane. This allows us to take rigorously into account the radial distortion of the image.

This paper is organized as follows. Section 2 describes briefly catadioptric image acquisition. The problem of catadioptric images anamorphosis will be discussed and a new adapted window for a pixel neighborhood will be derived under some theoretical considerations. An illustration of image matching using the new window on a real catadioptric image acquired with a paraboloidal mirror and CCD camera is presented in section 3.

2. CATADIOPTRIC VISION AND PROBLEM FORMULATION

This paper deals with catadioptric sensors that satisfy the single effective viewpoint (SVP) constraint [1]. This constraint is described in the following sub-section.

2.1. Catadioptric images formation model

The fixed viewpoint constraint is used so that only the intensity of light passing through this single point is measured by the camera [1][4]. If we assume, without loss of generality, that the viewpoint v of the catadioptric system lies at the origin of the Cartesian coordinate system, the effective pinhole is located at the point $p=(0,c)$ (see Figure 1 for an illustration), the $z-$axis $\hat{z}$ is in the direction vp and the used mirror is a surface of revolution along $\hat{z}$. The general equation of the fixed view point constraint is given by [4]:

$$r(c-2z)\left(\frac{dz}{dr}\right)^2-2\left(r^2+cz-z^2\right)\left(\frac{dz}{dr}\right)+r(2z-c)=0 \quad (1)$$

This equation is derived in the 2D Cartesian frame $(v,\hat{r},\hat{z})$ where $\hat{r}$ is the unit vector orthogonal to $\hat{z}$ and $\left(r=\sqrt{x^2+y^2},z\right)$ the two dimensional profile of the mirror $z(r)=z(x,y)$. The general solution of the fixed viewpoint constraint equation is:

$$\left(z-\frac{c}{2}\right)^2-r^2\left(\frac{k}{2}-1\right)=\frac{c^2}{4}\left(\frac{k-2}{k}\right) \qquad (k\geq 2) \qquad (2)$$

$$\left(z-\frac{c}{2}\right)^2+r^2\left(1+\frac{c^2}{2k}\right)=\left(\frac{2k+c^2}{4}\right) \qquad (k>0)$$

With k as the integration constant. The variations of c and k give the section of all convex mirrors. For instance, if $k=2$ and $c>0$ the resulting mirror is planar.

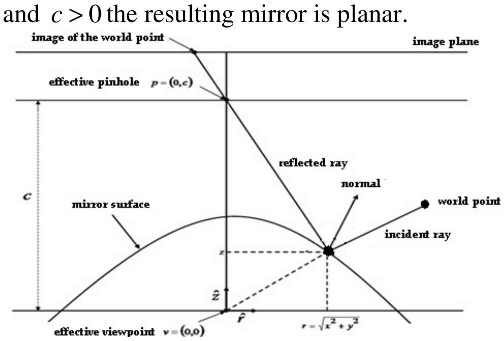


Figure 1: The geometry of image formation for catadioptric sensors with fixed viewpoint.

2.2. Anamorphosis and processing of catadioptric images

Images obtained by catadioptric sensors suffer from radial distortions, which render them complicate to interpret or to process [10]. In the classical images case, the set of image pixels is a regular sampling of the three-dimensional scene. The processing of those images can be done by a convolution of each pixel with a summative mask, which is generally square. The gray scale of the pixel is a combination of that pixel's value and its square neighborhood one. The mask is translation invariant and applied in the same way on the whole image.

In the catadioptric plane, a regular grid of the scene is entirely deformed by the convex mirror after projection. A straight line will be transform into a curve having the same curvature as the mirror. Two neighbor pixels on the catadioptric image do not belong any more to the same horizontal or vertical line, but lie on the same curve characterized by the mirror form; the square mask has no sense on this plane. The proposed window is firstly designed in the mirror and then projected onto the image plane. Two neighbors in the image are the projection of two neighbors in the mirror. This fact enforces the respect of the deformation caused by the mirror.

An arbitrary point on the mirror is defined from equation (2) by the vector:

$$X=\begin{pmatrix} x \\ y \\ z=f(x,y) \end{pmatrix} \qquad (3)$$

The final projection of the mirror point on the image plane can be written in the following general form [5], [9]:

$$\begin{pmatrix} u \\ v \end{pmatrix}=K_C R_C (X-T_C) \qquad (4)$$

Where (u,v) lies in the image plane, K_C is the matrix of the intrinsic parameters of the camera, R_C and T_C are respectively the rotation and the translation matrices between the mirror coordinates system and the camera system. These three matrices are estimated by a convenient calibration of the catadioptric sensor. The translation matrix is in all the cases defined by the vector $T_C=(0,0,-2e)^t$ ($e=\frac{c}{2}$ is the eccentricity of the mirror). For the parabolic mirror, the distance c tends toward infinity which leads to a parallel projection. The translation has no sense in this case; its vector is then equal to zero [9].

Since a convex mirror can be considered as a symmetric surface of revolution, the parameterization of that mirror is function of two parameters t and θ. The first refers to the z value of the mirror point. This value is the same over each circle belonging to the mirror. The second parameter is the angle of the progressive revolution going from 0 to 2π. The mirror point X will be parameterized by:

$$X=\begin{pmatrix} x(t,\theta) \\ y(t,\theta) \\ f(t,\theta) \end{pmatrix} \qquad (5)$$

The projection of this point on the catadioptric image plane is then:

$$\begin{pmatrix} u \\ v \end{pmatrix}=K_C R_C \left(X(t,\theta)-T_C\right)=\begin{pmatrix} u(t,\theta) \\ v(t,\theta) \end{pmatrix} \qquad (6)$$

Consequently for each (t,θ), we have (u,v) on the image plane. Furthermore, the displacement (h,k) of a pixel in the image plane is not $(u+h,v+k)$ any more as in the classical case but $(u,v)(t(i+h,j+k),\theta(i+h,j+k))$ where i,j are respectively the indices of the sampled t and θ. The projection of the pixel's neighborhood is illustrated in

Figure 2 for an example of orthographic projection.

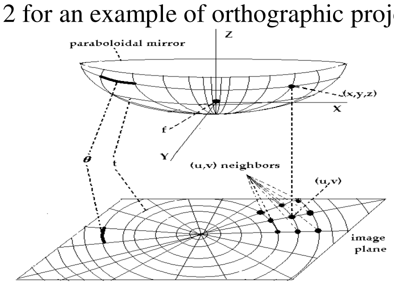
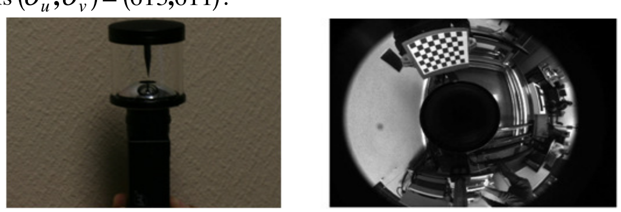


Figure 2: Sample of the parameterization of a catadioptric image using paraboloidal mirror.

As shown in this figure the projection of the mirror shape results in concentric circles of radius t and radials separated by the angles θ. This new window must be used for all processing algorithms in the catadioptric image plane. For instance, for Harris detector, one needs horizontal and vertical derivatives of the image. Using the adapted catadioptric mirrors, circular and radial derivatives will be introduced. The filter path will now look like a spiral from the center of the image toward its boundaries. The neighbors lie on the same circle t or on the same radial characterized by $tg(\theta)$.

3. APPLICATION

To achieve the previous theoretical considerations, experiments with real central catadioptric sensor with a paraboloidal mirror and a CCD camera are presented in this section. A telecentric lens is used to ensure a good orthographic projection in order to satisfy the SVP constraint. The calibration of the sensor with the method of Mei [7] gives the required parameters namely, the matrix K_C of intrinsic parameters of the camera and the focal f of the mirror. The translation is null in this case and as we assume the alignment of the optical axis and the symmetric axis of the camera, the rotation matrix is the unit matrix. Figure 3 shows the used sensor and the acquired image including the grid used for the calibration process. The image size is 1220×1620. The intrinsic matrix of the camera is an upper triangular matrix. Its parameters are estimated as:
1) the projection of the camera center on the image plane is estimated to be at the pixel $(u_0, v_0) = (661,736)$.
2) The resolution of the pixel multiplied by the focal mirror is $(\sigma_u, \sigma_v) = (613,611)$.



(a) (b)

Figure 3: (a) Catadioptric sensor with paraboloidal mirror and telecentric lens. (b) Catadioptric image acquired with the sensor (a).

1	1	1	1	1
1	1	1	1	1
1	1	**1**	1	1
1	1	1	1	1
1	1	1	1	1

(a)

1	0	0	0	0
1	1	1	0	0
1	1	**1**	1	1
0	1	1	1	1
0	0	0	1	1

(b)

Figure 4: (a). Classical mask. (b). Catadioptric mask. The concerned pixel is in a bold font.

The equation of the paraboloidal mirror of parameter f positioned under the camera as in Figure 1 is given by:

$$z = -f + \frac{x^2 + y^2}{4f} \qquad (7)$$

There are several ways to parameterize a surface. For these experiments, we chose the one using the mirror parameter f, the concentric circles t and the angles of the radials θ. For a mirror point (x, y, z) on the mirror, its parameterization is:

$$x = f\sqrt{2t}\,cos(\theta), \quad y = f\sqrt{2t}\,sin(\theta), \quad z = f \times (t-1) \qquad (8)$$

Where $\theta \in [0, 2\pi[$ and $t \in [0, h]$. h is the distance between the vertex and the boundary of the mirror.
The projection of the parameterized mirror point onto the image plane results in a pixel which is function of mirror parameters. Thus, using the projection model of equation (4) and the parameterization of equation (8) the point (u, v) on the catadioptric image plane will be:

$$u = u_0 + \sigma_u \sqrt{2t}\,cos\left(\theta + \frac{\pi}{2}\right) \qquad (9)$$
$$v = v_0 + \sigma_v \sqrt{2t}\,sin\left(\theta + \frac{\pi}{2}\right)$$

The pixel is designed to be the intersection of horizontal and vertical lines of the camera matrix. However with the new model, the pixel will be defined as (u, v) and its neighbors are positioned in the same way as illustrated in Figure 2. In fact, two neighbors lie on the same circle of radius $\left(\frac{u-u_0}{2\sigma_u}\right)^2 + \left(\frac{v-v_0}{2\sigma_v}\right)^2$ or on the same radial of angle $tg^{-1}\left(\frac{\sigma_u}{\sigma_v}\frac{v-v_0}{u-u_0}\right)$.

Figure 4 shows a comparison between the classical and the adapted 5×5 mask surrounding the central pixel located at the coordinates $(50,590)$ in Figure 3b. In order to detect interest points in the image for the matching process, Harris corner detector is chosen. It is based on image derivatives and the convolution with the Gaussian kernel. Traditionally the derivation is in the discrete image support as the difference of its horizontal or vertical lines. Instead, with the new sampling the circular and radial derivatives of the image I are used and defined as:

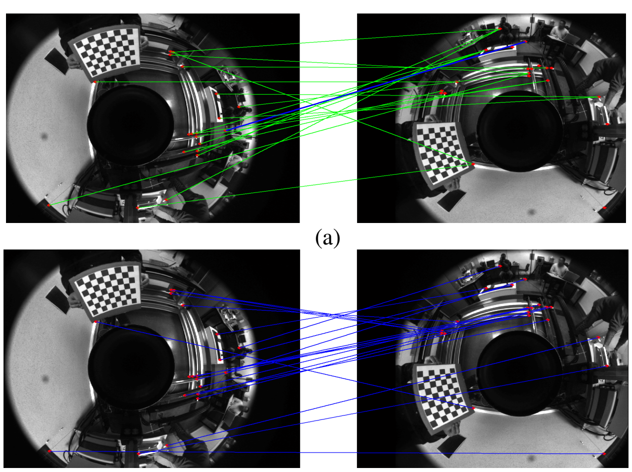
$$\frac{dI}{dt} = \frac{\partial I}{\partial u}\frac{\partial u}{dt} + \frac{\partial I}{\partial v}\frac{\partial v}{dt} \qquad (10)$$
$$\frac{dI}{d\theta} = \frac{\partial I}{\partial u}\frac{\partial u}{d\theta} + \frac{\partial I}{\partial v}\frac{\partial v}{d\theta}$$

$\frac{\partial I}{\partial u}$ and $\frac{\partial I}{\partial v}$ refer to the difference of the circles and radials in the image. The convolution and the determination of the Gaussian will be defined in the same way.

For our experiment, the pair of catadioptric images is taken with the parabolic sensor. Considering the left image as the reference, the right image is acquired after the motion of the sensor along its optical axis. This motion combines a rotation estimated to 79.5° and a translation of 2cm. Harris algorithm is used to detect 20 interest points. The dissimilarities between the detected primitives in the pair of images are computed by using the NSSD (Normalized Sum of Squared Differences) function defined by:

$$NSSD(u,v) = \frac{\| neigh(u) - neigh(v) \|^2}{\sqrt{\| neigh(u) \| \| neigh(v) \|}} \qquad (11)$$

Where $neigh(u)$ and $neigh(v)$ are the circular and radial neighborhood of pixels u in the first image and v in the second image as denoted in Figure 2. These two windows forms (classical and adapted) are applied for Harris detector and matching tasks. Figure 5.a shows the result of the application of the classical matching algorithm with square neighborhood and Figure 5b illustrates the result of the new method. The difference is well visible on the two images. In the classical case, all of the interest points are badly matched (green lines) except for one pair of pixels (in blue). The main reason is the erroneous



(a)

(b)

Figure 5: Matching of catadioptric images with: (a) Classical method. (b) Adapted method. The motion of the sensor is a rotation (79.5°) and a small translation (2cm) around its optical axis (Blue for well matched pixels, green otherwise).

neighborhood of the pixel. In the new method all of the pixels are one to one matched. The detection of interest points does not indicate outliers points which appear in one image and not on another. In this part of work, we did not deal with this question. This problem will be treated in future works to perform dense correspondence of catadioptric images.

4. CONCLUSION AND FUTURE WORKS

In this paper, a new method to process catadioptric images is derived. This method is based on an adapted neighborhood of the pixel. This neighborhood is designed as the projection of the window surrounding the pixel in the used mirror after its parameterization. This new technique leads to an effective matching of catadioptric images comparing with the classical method. The perspective in relation with this work is the dense correspondence of catadioptric images using this new formulation of the pixel's neighborhood.

5. REFERENCES

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