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Modelling agronomic images for weed detection and comparison of crop/weed discrimination algorithm performance

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Abstract A new method for weed detection based on modelling agronomic images taken from a virtual camera placed in a virtual field is proposed. The aim was to measure and compare the effectiveness of the developed algorithms. Two sets of images with and without perspective effects were simulated. For images with no perspective, based on Gabor filtering and on the Hough transform, the performance of two crop/inter-row weed discrimination algorithms were tested and compared. The method based on the Hough transform is, in any case, better than the one based on Gabor filtering. For images with perspective effects only, an algorithm based on the Hough transform was tested and an extension to real images is discussed. These tests were done by a comparison between the weed infestation rate detected by these algorithms and the true one. This evaluation was completed with a crop/weed pixel classification and it demonstrated that the algorithm based on a Hough transform gave the best results (up to 90%).

Keywords Simulated images · Spatial statistics · Weed infestation · Hough transform · Vanishing point · Gabor filter

Introduction

For site-specific weed management, many online systems using different optical sensors have been developed to enable spraying specifically the weed-infested areas (Felton and McCloy 1992; Felton 1995; Tian et al. 1999). In this context, an efficient image processing procedure for crop/weed discrimination is required in order to quantify weed infestation rates but a manual evaluation of the weed infestation rate (WIR) is a difficult task: it will

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take a very long time for either a manual segmentation of the image or a manual counting of weed plants in the field. Very few articles have reported on the evaluation of the robustness of crop/weed discrimination algorithms which have actually been validated from real images with natural weed patterns taken from a camera under natural outdoor lighting conditions (Andreasen et al. 1997; Tang et al. 1999; Onyango and Marchant 2005). Some algorithms have been developed in our lab and have been tested on real data and in real in-field conditions but assessing and comparing them appeared difficult and uncertain (Vioix et al. 2002; Bossu et al. 2006).

We have developed a new and original method dedicated to site-specific weed management. We propose to model photographs taken from a virtual camera placed in a virtual crop field with different common weed infestation rate. In fact, a simulated image under various conditions, with knowledge of every parameter (weed and crop pixel number and position) is a perfect tool for evaluating the accuracy of any algorithms for discriminating between crop and weed.

The purpose of this article is to present a simulation of agronomic images generated from a virtual field composed both of crop and weed plants represented by different patterns. Simple statistical models of spatial distributions of weed species in a crop field were developed to generate the virtual field considered as a two-dimensional surface. Afterwards, to mimic pictures taken from a camera, a pinhole camera model is used; it allows simulations of any possible configuration (camera characteristics, position and orientation). In order to evaluate the accuracy of a crop/weed discrimination algorithm, image parameters (number of crop pixel, weed pixel and weed infestation rate) have to be saved for comparison with those detected by the algorithms.

In this article, we propose an application of this modelling dedicated to images without perspective and assess the effectiveness of two crop/inter-row weed detection algorithms based on Gabor filtering (Vioix et al. 2002) or Hough transform (Jones et al. 2007a, b). Then, for images with perspective effects, a crop/weed discrimination algorithm based on the Hough transform was analysed and tested on real images.

For each algorithm, the accuracy of $WIR_{\text{inter-row}}$ for each image was calculated by comparing it directly to the true $WIR_{\text{inter-row}}$ fixed during the simulation of images. An analysis of algorithm performance was carried out and the plant discrimination results are presented. As a by-product of this simulation, an improvement of the discrimination algorithm was proposed and discussed. Finally, we propose improving the possibilities of this modelling considering a spectral approach. It would be a way to evaluate both spatial and spectral algorithms for weed detection.

Materials

A set of agronomic images was simulated to test and validate the effectiveness of any crop/weed discrimination algorithm. Before modelling images taken from a virtual camera, a model of a crop field infested by weeds is required. Then, a modelling of images taken from a virtual camera placed in a virtual field was carried out.

Virtual field

To model the spatial distribution of crops and weeds in a virtual field, we chose very simple mathematical models considering the field as a black and white two-dimensional surface. Homogeneous soil is considered, as a first approximation to be black. Crop is sown in lines spaced by a constant width (the inter-row width) depending on the crop type (wheat: 160–180 mm, sunflower: 450 mm…). Two different sowings are proposed: one corresponding to cereal sowing pattern (only an inter-row width is pre-defined) and the other corresponding to sunflower (or maize) sowing pattern (an inter and intra-row is pre-defined). To simply model the 3-dimensional reality, for each type of crop, different patterns are available and can take different orientations and different sizes. Crop presence is controlled by a stochastic variable to reproduce growth issues and to render the unpredictable nature of plant growing.

An example of a wheat field modelling is presented in Fig. 1 with a true WIR of 20%. The true WIR is composed of a $WIR_{inter\text{-}row}$ of 18.6% and a $WIR_{intra\text{-}row}$ of 1.6%.

In the case of weed plants, it is established that three weed spatial distributions are usually observed in crop field: isolated, aggregative or a mixture of both (Cardina et al. 1997; Van Groenendael 1988). In both cases, the weed plant number is a countable number so that we used discrete spatial distributions for modelling. For isolated weed plants, a Poisson process is applied whereas in the case of a weed patchy distribution, we used a Neyman-Scott process based on a Poisson process. These probability distributions have been developed by other authors (Miles 1970; Fisher and Miles 1973) to model plant population dynamics. So three different stochastic processes were used: the Poisson law, the Neyman-Scott aggregative process and a mixture of both depending on the weed plants present in the field. Moreover, for this modelling, we take into account French agricultural practice where the first herbicide treatments are applied at an early crop growth stage when the weed plant population is low. The farmer usually implements tillage practices before sowing with different mechanical methods (tillage equipment: plough or cover-crop) in order to avoid germination and growth of weeds. Consequently, the number of weeds could be considered as very low compared to crop ones.

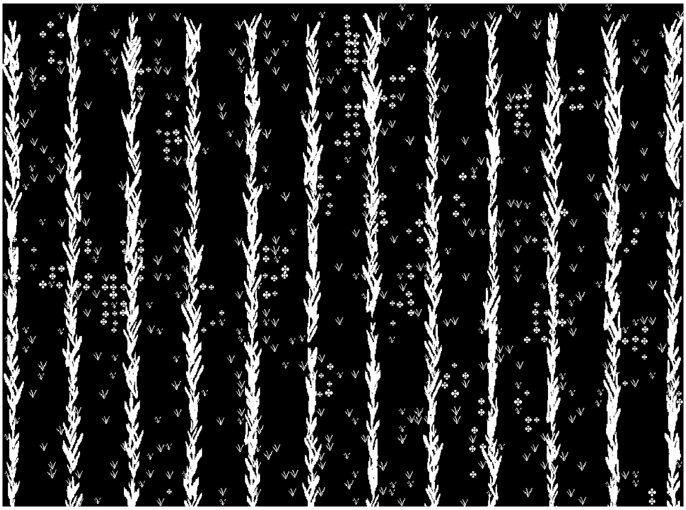

Fig. 1 A virtual wheat field (inter-row spacing: 16 cm). The weed spatial distribution is a mixture of both weed spatial distributions, isolated and clustered. The true WIR is 20% with a true $WIR_{inter\text{-}row}$ of 18.4% and a true $WIR_{intra\text{-}row}$ of 1.6%

Poisson process

We attempted to model a crop field at a specific time without any knowledge of the history of the field parameters (soil, plant seed reproduction, farming practices). Assuming that the weed spatial distribution is a random process with no memory between successive events and that occurrence of the emergence of weed plants compared to crop plant is very low, it can be fully modelled by a Poisson punctual process (Goreaud 2000; Miles 1970). Then, in order to form an important statistical set of data, we subdivided the global field surface (D) of the image into a set of small areas (S). Assuming that all the events occurring in one area are independent of those occurring in another area and assuming that the mean weed density in these areas is λ as in the entire field, the probability of emergence of a number of k weed plants in an area can be expressed by a Poisson distribution with parameter equal to λS defined as follows:

$$P_k(\lambda S) = P(X = k) = \frac{(\lambda S)^k}{k!} \mathrm{e}^{-\lambda S} \tag{1}$$

where X is the stochastic variable associated with the process.

Afterwards, the number (k) of weeds in small areas is randomly determined by applying a discrete random number generator (Ripley 1983). Then, the positions (x, y) of the k weeds are randomly chosen using a uniform distribution restricted to the image size.

Neyman-Scott process

The Neyman-Scott process is suitable for modelling clustered populations (Fisher and Miles 1973). The Poisson process is complemented by a Neyman-Scott aggregative process to generate a more realistic spatial distribution of weeds that is usually patchy in a cereal field (Brown and Steckler 1995). We assume that the growth of weed patches is restricted on a pre-defined ellipsoid region ($S_{ellipsoid}$) depending on the cultivation practices (e.g. ploughing) which tend to elongate the weed distribution along the crop row direction. The restricted area is defined as:

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} < 1 \tag{2}$$

where a, b are the radii of the ellipsoid in the x and y directions.

Firstly the number of patches and then the number of weed plants per patch were determined.

A mixture of both distributions has been also considered to create fields as realistic as possible. A virtual wheat field is presented in Fig. 1 simulating a cereal crop field (the intra-row frequency is 160 mm). From this image, the true weed infestation rate (true WIR) is determined by:

$$\text{true WIR } (\%) = \frac{\text{weed pixels} * 100}{\text{(crop + weed) pixels}} \tag{3}$$

and the true WIR is composed of:

$$\text{true WIR}(\%) = \text{true WIR}_{\text{inter-row}} + \text{true WIR}_{\text{intra-row}} \tag{4}$$

where

$$\text{true WIR}_{\text{inter-row}}\ (\%\ ) = \frac{\text{inter-row weed pixels} * 100}{(\text{crop} + \text{inter-row weed})\ \text{pixels}} \tag{5}$$

Virtual photographs

A virtual camera (CCD height: Hccd = 5.28 mm and CCD width: Lccd = 7 mm; focal length: $f = 16$ mm) is located in the field with 2 degrees of freedom (pitch angle, tilt angle); the roll angle is not considered. From the pinhole camera model (Faugeras 1993), we are able to map the real world co-ordinates of a point into its pixel co-ordinates in the image space. The field image size has to be included in the camera's field of view and consequently the field size is given by the smaller rectangle containing the CCD projection. The difference of resolution between the virtual field and the CCD of the virtual camera implies a mixture of several pixels for each image pixel during the projection transformation. Particularly, at the border of vegetation leaf and soil, a pixel can be composed of both type of pixel. The resulting image is a grey-scale image. Thus, this new virtual image is obtained as presented in Fig. 2.

Image database

31 Series of 50 images were constructed in order to test the efficiency of the crop/weed algorithms; for each series, the true WIR was fixed from 0 to 60% with a 2% step. For each of the three weed distributions, 4,650 images were analysed.

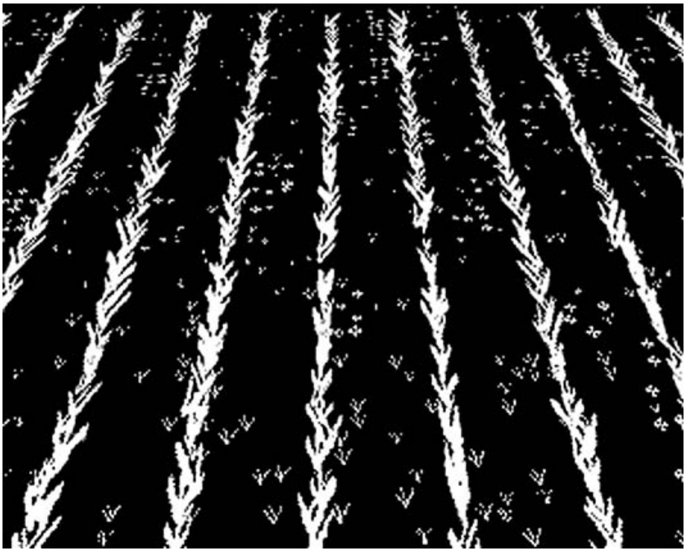

Fig. 2 Virtual wheat image: inter-row width = 16 cm, camera's height = 1 m, pitch angle = 50° and a true WIR = 20% with a mixture of both weed spatial distributions

Methods

Two different algorithms for crop/weed discrimination have been tested and compared. For both algorithms, their discrimination depends on spatial information and consequently only $WIR_{inter\text{-}row}$ can be detected. For images without perspective effects, a Gabor filtering (Gabor 1946) and an algorithm based on the Hough transformation (Hough 1962) have been tested whereas in the case of an image with perspective effects only the Hough Transform has been studied.

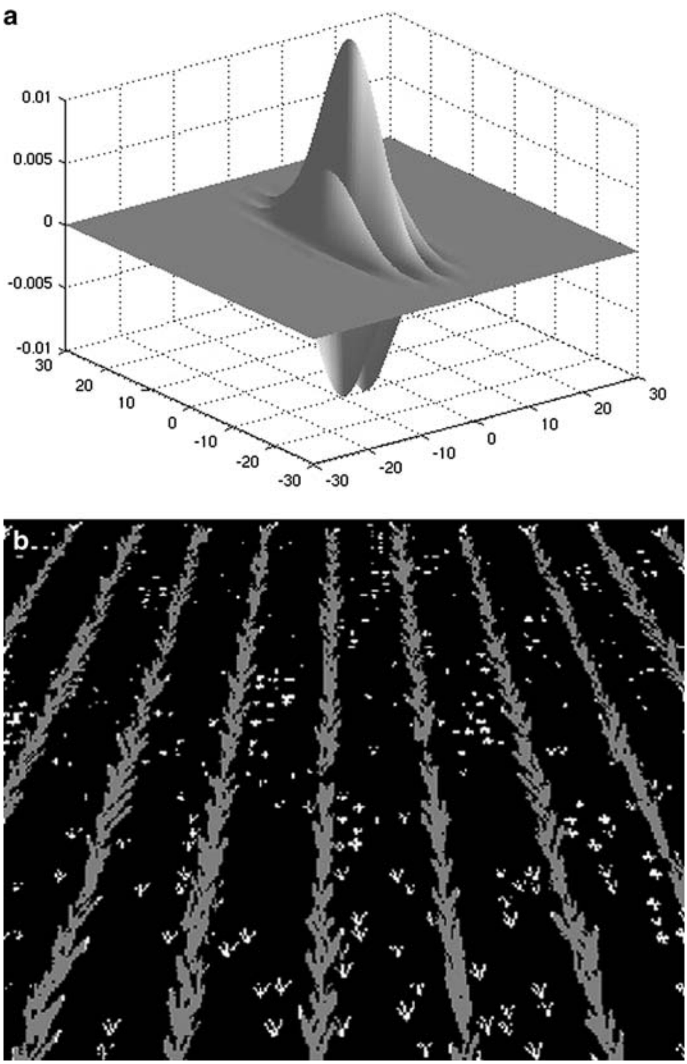

Fig. 3 **a** Spatial representation of Gabor filter. **b** Crop/weed discrimination with a Gabor filtering. Pixels assigned as crop are in grey and pixels assigned as weed are in white

Gabor filtering

A Fourier transform operation (frequency domain conversion) was tested according to the fact that crops are associated with repetitive structures with a frequency close to ω. We apply a real bi-dimensional Gabor filter defined as a modulation of a Gaussian function by a cosine signal:

$$g(x,y) = \frac{1}{\pi \times \sigma_x \times \sigma_y} \times \exp\left(-\frac{x^2}{2 \times \sigma_x^2} - \frac{y^2}{2 \times \sigma_y^2}\right) \times \cos(2 \times \pi \times \omega \times x) \qquad (6)$$

It is centred on the frequency ω; σ_x and σ_y set the bandwidths, respectively along the x and y axis. The spatial representation of this filter *g(x,y)* is shown in Fig. 3a. Then from a simple logical inhibition function (AND with an inverse input) between the filtered image and the inverse vegetation binarized image, an inter-row weed map is deduced. A detailed explanation can be found in Bossu et al. (2006) or Vioix et al. (2004). The crop/weed discrimination is presented on Fig. 3b.

Hough transform

Hough transformation is a technique which can be used to isolate features of a particular shape within an image. The crop/weed discrimination algorithm is firstly based on crop row detection from the vanishing point (Hough Transform) and secondly

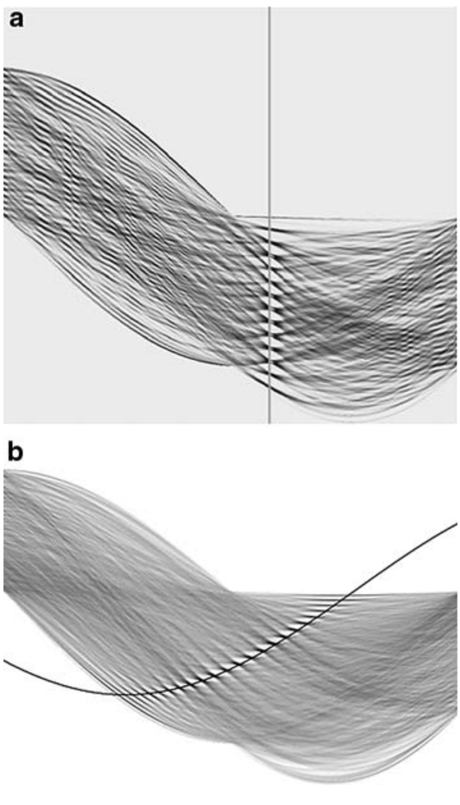

Fig. 4 **a** Image with no perspective effects—in Hough space: detection of the main maxima belonging to the line associated with the vanishing point. **b** Image with perspective effects—in Hough space: detection of the main maxima belonging to the sinusoid curve associated with the vanishing point

based on crop/weed segmentation (region-based segmentation) for crop/weed pixel classification.

To generate the Hough transform (Hough 1962) for matching lines in the image, the two polar parameters (the radius ρ and the angle θ) (Duda and Hart 1972) were chosen, rather than slope-intercept, where ρ is the perpendicular distance from the image origin to the line and θ is the angle between the normal and the X-axis. Then for each angle value, the line parameterized equation $\rho = X \times \cos\theta + Y \times \sin\theta$ is solved and these parameters were collected in a two-dimensional accumulator array, H(ρ, θ). A normalization of the accumulator is also performed in order to promote lines positioned close to the borders of

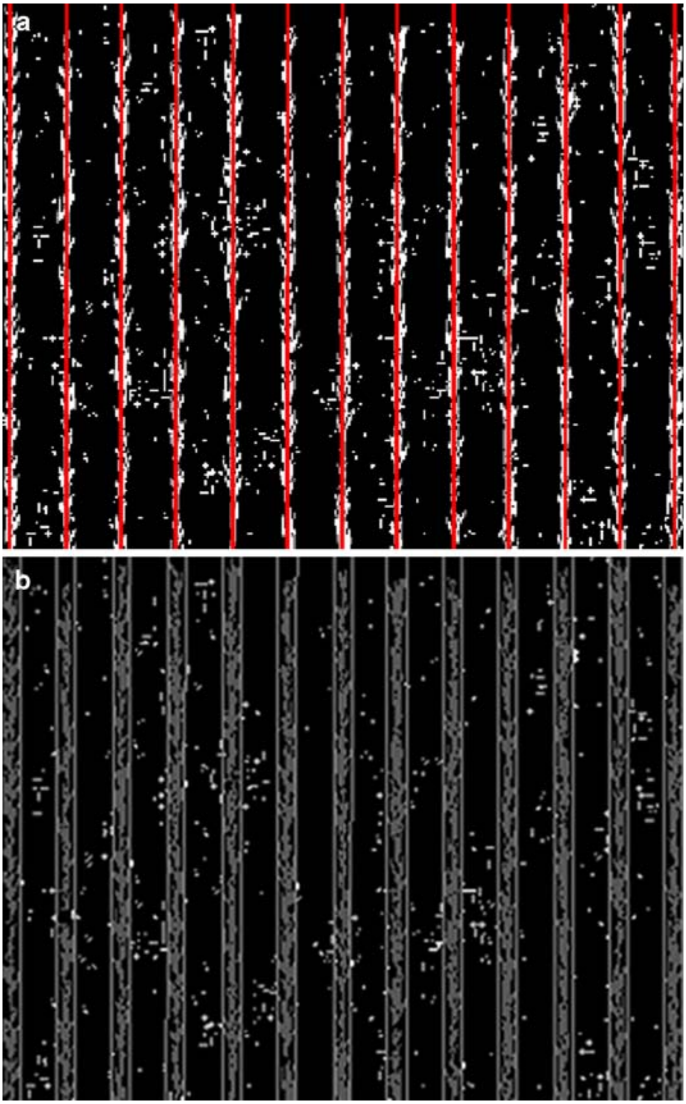

Fig. 5 **a** Image with no perspective effects—crop row detection in a wheat field. **b** Image with no perspective effects—crop/weed discrimination in a wheat field

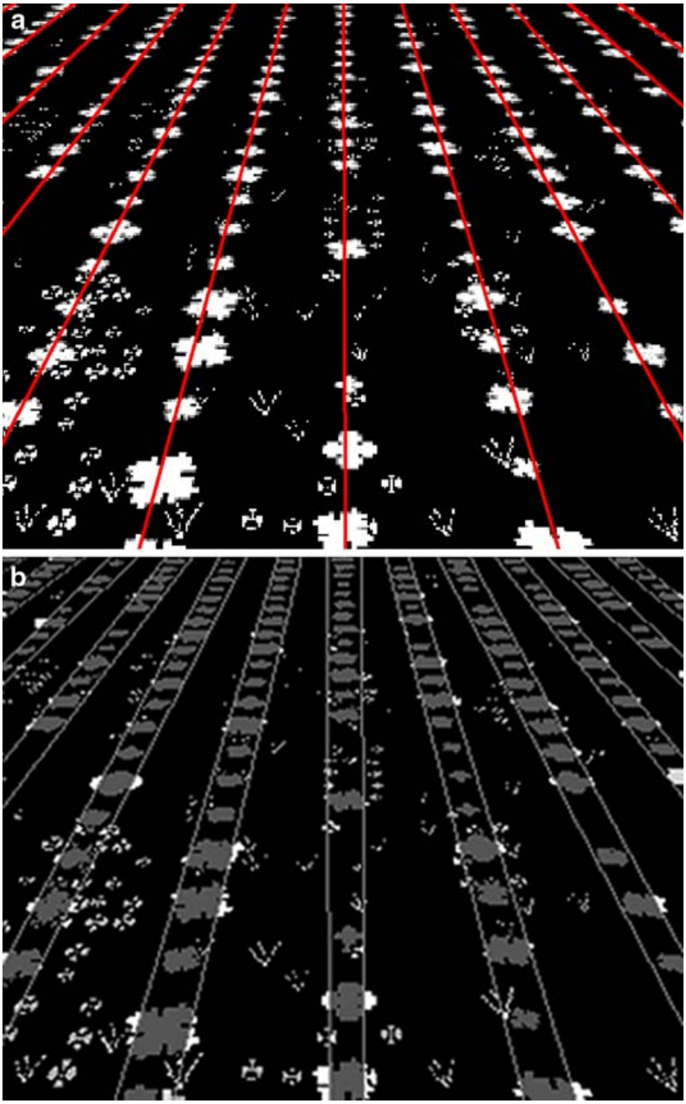

Fig. 6 **a** Image with perspective effects—crop row detection in a sunflower field. **b** Image with perspective effects—crop/weed discrimination in a sunflower field

the image. It is defined as the ratio between the accumulator of the vegetation image and the accumulator of a white image of the same size.

Thus, the problem of line detection in an image becomes simple local peak detection in the Hough space and we developed an algorithm based on the vanishing point detection. Depending on the perspective effects or not in the image, two algorithms were developed.

For images with no perspective effects, the scene represents a set of parallel crop rows with a constant spacing where the vanishing point is infinite. The global maximum, (ρ_m, θ_m) is usually equivalent to the most intense crop row in the image space and its detection is an easy way to have information about all crop rows (i.e. ρ_{lm} and θ_{lm}) in the image. According to the fact that crop rows are parallel in the image, the consequence in Hough space is that all their local maxima will be aligned on the global maximum and will have the same θ-value as the global maximum (i.e. $\theta_m = \theta_{lm}$). A threshold of the line profile gives us all the maxima (Fig. 4a).

For images with perspective effects, all maxima associated with crop rows will belong to a sinusoid curve, associated with the vanishing point. The line extraction procedure is based on different automatic thresholds (Fig. 4b) which are fully described in Jones et al. (2007a).

Then, for both cases (image with and without perspective effects), all the detected lines (Figs. 5a and 6a) are labelled as “crop”.

The second step of the algorithm was the discrimination between crop and inter-row weed pixels. Applying a blob colouring analysis, all contiguous pixels of the same colour intensity were grouped into the same region. Afterwards, a two-class (crop/weed) pixel classification is done according to the fact that if a pixel of a straight line belongs to a region then the region is labelled as “crop” otherwise it is labelled as “weed”. However when weed pixels are close to crop pixels, they are grouped into the same region labelled as “crop” causing some misclassifications. Therefore we optimized the method by fitting each border of the crop region with a simple straight line as shown in Figs. 5b and 6b.

Results and discussion

Images with no perspective effects

From Figs. 3b and 5b, the comparison between the true $WIR_{inter\text{-}row}$ and the detected $WIR_{inter\text{-}row}$ demonstrates that the classification method leads to misclassification errors. To understand these errors and to evaluate the accuracy of this method, we summarize the classification results in a confusion matrix which indicates the number of correctly and incorrectly classified pixels (both weed and crop classes). Figure 7 presents crop/weed result for Gabor filtering and Hough transformation respectively for images with a true WIR of 20%. Figure 8 shows the classification results with a true WIR of 60%. Both algorithms give good classification results (up to 75%). The crop/weed discrimination algorithm based on the Hough transform gives more accurate results, up to 88% with a higher computation time.

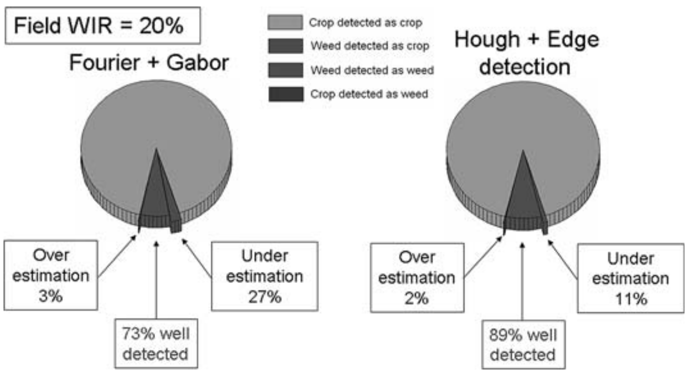

Fig. 7 Results of crop/weed classification for both Gabor and Hough algorithms for image with a true WIR of 20%

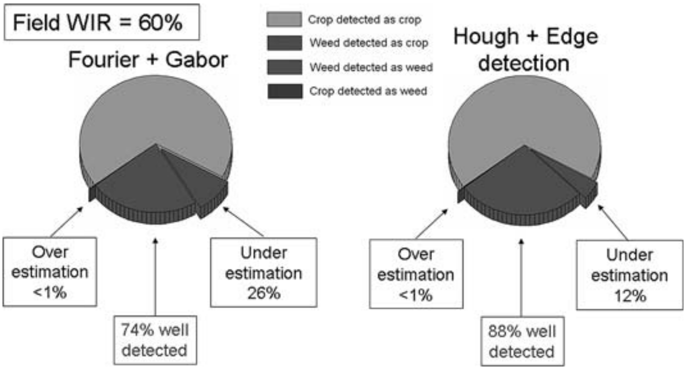

Fig. 8 Results of crop/weed classification for both Gabor and Hough algorithms for image with a true WIR of 60%

With the Gabor filtering, it was noted that there were a significant number of weed pixels which were always considered as crop pixels implying an underestimation of the weed detection. This is probably due to the fact that the bandwidth value of the Gabor filter automatically detected in Fourier space is over-estimated. Consequently, all weed pixels close to crop row are classified as crop pixels. In conclusion, whatever the true WIR (20 or 60%), these algorithms seem to be also reliable in the presence of high weed infestation.

Images with perspective effects

Results of the crop/weed discrimination algorithm are presented in Fig. 9. With regard to the image of Fig. 2, the detected inter-row $WIR_{inter\text{-}row}$ was 16.6% whereas the true $WIR_{inter\text{-}row}$ was 18.4% in the image of Fig. 1. From the confusion matrix, the high degree

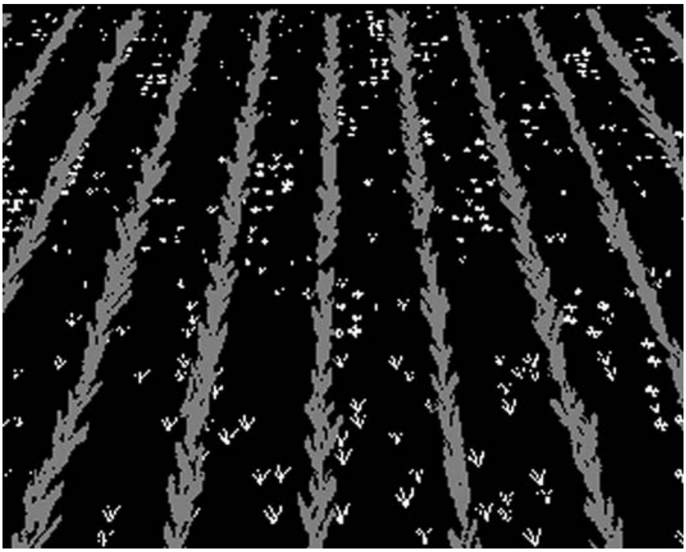

Fig. 9 Results of crop/weed discrimination from Fig. 1. The detected $WIR_{inter\text{-}row}$ was 16.6%

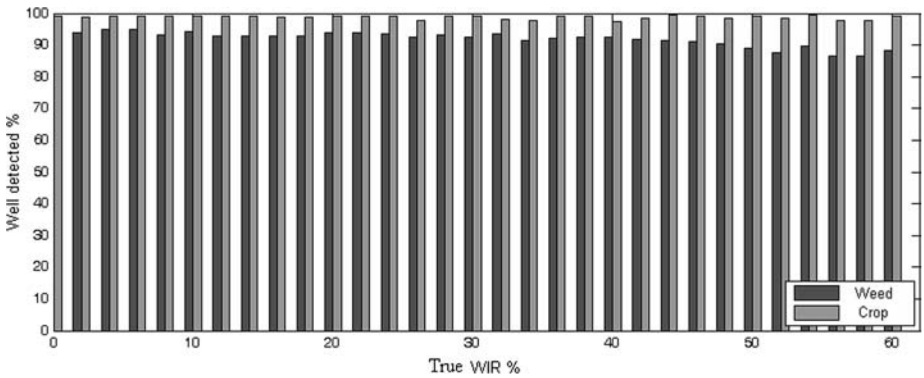

Fig. 10 Results of misclassification of weed pixels for the classifier on a virtual image database with mixture spatial distribution of weed plants according to the camera parameters of the image of Fig. 2. Correct crop/weed pixel classification depending on the true WIR (%) is in grey/black

of accuracy (up to 90%) indicates the very good performance of this algorithm in discriminating between crop and weed pixels.

To assess the effectiveness of this algorithm in the presence of noise (i.e. weed), we attempted to analyse its behaviour with images of different series with different true WIRs. Consequently, this algorithm has been tested on a database of 31 series of true WIR, each one composed of 50 images in order to have a statistically significant group. The results are presented in Fig. 10 in the case of a weed distribution composed of a mixture of Poisson and Neyman-Scott processes. The classification method gives coherent results indicating its robustness (up to 90%) for low, medium or high true WIR. In these conditions, we confirm the good results obtained for the overall accuracy of the algorithm.

For perspective images, as only one algorithm has been tested, we chose to apply the simulation-based results for predicting performance in real field conditions. In this case, a $WIR_{inter\text{-}row}$ can be estimated from the algorithms on natural images. An example is

Fig. 11 Result of crop/weed discrimination algorithm (Hough transform, blob colouring + crop edge detection) on a natural image. The detected $WIR_{inter\text{-}row}$ is 31% whereas the true WIR is 28.4%

presented in Fig. 11 where the detected $WIR_{inter\text{-}row}$ is 31%. However, it is quite complex to estimate the accuracy of this result. Indeed, a comparison with ground truth would have been possible but the manual work to count the weed plants in the field of view of each image and then to transpose the weed plants into weed pixels would have been too intense. An alternative (and also intense) method based on a manual segmentation of the images has been used. In this case, the true $WIR_{inter\text{-}row}$ has been estimated at 28.4%. However, more data images should be investigated in order to test the algorithm performance on real images.

Evolution of the modelling

At the present time, the modelling can only be used to test spatial crop/weed discrimination algorithms. The algorithms based on plant shapes or other plant features cannot be tested because the plant patterns in this modelling are just used to provide a better similarity with nature but it was not an essential part.

This modelling seems to be also a good way to compare and assess the accuracy of any algorithms to discriminate between crop and weed.

Also the present version of this model could be a helpful tool to test and optimize the precision of some agricultural implements based on vision systems. Indeed, in the context of reduction of herbicides, some researchers (Tillet et al. 1998; Åstrand and Baerveldt 2002) investigated the development of mechanical weed control techniques based on vision in order to remove single weed plants localized in the crop row. However during the trial period of these tools, extensive tests must be done either outdoors in real fields depending on weather conditions, the stage of plant growth or indoors positioning artificial plants in a corridor. Implementing simulated agronomic images in the computer which controls the tool (i.e. cycloid hoe, rotary-tine weeder, rotary hoe…), it becomes an easy way to test it without spending a lot of time to prepare and manage experimental crop fields over the experiment periods.

Currently the virtual images are in grey-scale and, to be more realistic, this modelling can be improved by investigating spectral properties of vegetation. A projection model of an incident light (i.e. sun) on the virtual field in order to simulate the reflection of plant leaves depending on their location can be implemented. For instance, some authors report different modelling of radiation propagation and absorption in plant leaves in order to determine their spectral signatures (Jacquemoud and Baret 1990). A radiative transfer model such as the one developed by Hapke (1981), can be also used to approximate the spectral signature of soil in different configurations.

Conclusion

The aim of this study was to propose a spatial modelling of crop/weed images to assess the robustness of any algorithms from simulated images whatever the crop type and whatever the camera location in the field. A first application of this modelling of agronomic images has been proposed for ranking different weed detection methods. Two crop/inter-row weed discrimination algorithms were tested especially dedicated to images with no perspective effects: a Gabor filtering and a Hough transform combined with a region-based segmentation. The comparison of the crop/weed pixel classification results indicated that the weed detection accuracy of the algorithm based on Hough transform is better than 90% on simulated images whereas, with the Gabor filtering, it is up to 75%. Consequently, the

Hough transform algorithm is more robust and accurate than Gabor filtering. Another application of this modelling has been to test a crop/weed discrimination algorithm based on Hough transform dedicated to images with perspective effects. In this case, we also observed a very high degree (up to 90%) of accuracy for this algorithm.

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