SALIENT POINT CHARACTERIZATION FOR LOW RESOLUTION MESHES

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ABSTRACT

This paper deals with edge detection and vertex characterization of 3D meshes. First, a new measure is introduced, the saliency degree, which provide directly the shape information of a vertex of the mesh without any curvature computation. The relationship between the saliency degree and the curvature is demonstrated for continuous and discontinuous 2D curves. Then, a description of the 3D operator is given. Finally, results on geometric and real objects are shown. The contributions of this work are the definition of a new 3D operator which is able to extract point characteristic on 3D low resolution mesh.

***Index Terms*—** 3D Salient point detection, saliency degree measurement, point/vertex characterization, SUSAN 3D

1. INTRODUCTION

Nowadays, one of the most important research topics, in the broad domain of 3D data analysis, is the characterization of 3D points. Indeed, as expressed Y.K. Lai et al. [5]: “Sharp edges, ridges, valleys, and prongs are critical for the appearance and accurate representation of a 3D model.” Most recent techniques are based on curvature analysis, and applied to different types of meshes. Based on the work and state of the art established by D.L. Page et al. [8], two operator classes can be summarized:

- Operators based on curvature analysis classified by D.L. Page et al. in three types, SFM [11], TCM [2] and CFM [15].
- 2D operators extended to 3D data or operators that adapt the 3D data for 2D operators.

On not sufficiently smooth meshes, or in the presence of a discontinuity, operators based on curvature analysis can not compute the curvature, because of its definition. Y.K. Lai et al. [5], D.L. Page et al. [8], M. Desbrun et al. [2] and others assume that the surface is sufficiently smoothed to apply theirs operators. Moreover, the presence of a sharp edge often infers additional computations. The method presented in this article belongs to the second class of operators, and propose an extension of the 2D SUSAN operator for regular or irregular triangle meshes. Our goal is to define a 3D operator which classifies the different types of vertex, on a simplified or a not very dense mesh, without any curvature computation. The extracted characteristics as curvature can be used for mesh simplification, segmentation or denoising.
First, a brief recall of the 2D SUSAN operator is presented and the relationship between the SUSAN response and the curvature established. Then, the 3D SUSAN operator will be described and some restrictions of the operator established. Results on geometric and real objects are shown. Finally, we conclude and give perspectives for future works.

2. PRINCIPLE OF THE OPERATOR

2.1. Recall on the 2D SUSAN operator

2D SUSAN operator described by S.M. Smith et al. [13] is a circular pixellized window in which the pixel under analysis is named nucleus. A reference attribute is chosen for the nucleus, for example the gray level for 2D images. In the circular window, the USAN area is defined as the set of pixels presenting a close value (bounded by a threshold) to the nucleus attribute. The operator response is the ratio of the USAN area over the total area of the circular window. In this way, a salient point corresponds to a minimum response of the SUSAN operator.

2.2. Relation between the saliency degree and the curvature

During the inspection of an object, the information of the presence of an edge is extracted by the SUSAN operator response. On an edge of an object, which is enhanced enough to clearly differentiate it from the background, and for an adapted analysis window, the SUSAN response depends on the curve of the object. The SUSAN response value, named saliency degree, depends on the curvature value. To define this relationship, the edge of the object is considered as a 2D parabolic function:

$$f(x) = ax^2 \qquad (1)$$

The curvature of a 2D function is defined by:

$$\kappa(x) = \frac{f''(x)}{(1+f'(x)^2)^{3/2}} \qquad (2)$$

The nucleus of the operator is centered on the point C_0 at the coordinates (0). At this point, the curvature is equal to:

$$\kappa(0) = 2a \qquad (3)$$

As presented in Figure 1, for $a<0$, the USAN area corresponds to the surface of the intersection between the circular window and the curve (in gray in the figure).

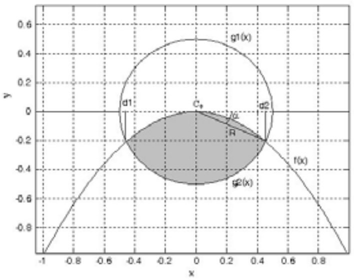

Fig. 1. Intersection of a circular window and a sufficiently smooth surface with $f(x)=-x^2$ and R=0,5.

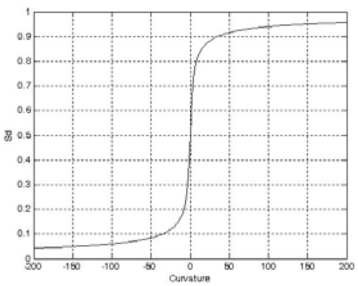

Fig. 2. Evolution of the saliency degree as a function of the curvature.

We define d_1 and d_2 as the x coordinates of the intersection between the circular window and the *f(x)* function:

$$d_1 = -\sqrt{\frac{\sqrt{1+4a^2R^2}-1}{(2a^2)}} = -d_2 \qquad (4)$$

We can determine the value of the USAN area A_{USAN} as follow:

$$A_{USAN} = \int_{-R}^{R} g_2(x)dx - \int_{-R}^{d_1} g_2(x)dx - \left|\int_{d_1}^{d_2} f(x)dx\right| - \int_{d_2}^{R} g_2(x)dx$$

$$A_{USAN} = \frac{\pi R^2}{2} - \frac{|a|(d_2{}^3 - d_1{}^3)}{3} - R^2\left(\alpha - \frac{\sin(2\alpha)}{2}\right)$$

Where α corresponds to the angle presented in Figure 1 and which is defined by:

$$\alpha = \arccos\left(\frac{d_2}{R}\right) = \arctan(ad_2) \qquad (5)$$

Using the value of A_{USAN} we define the salient degree S_d at the point C_0 as:

$$S_d = \frac{A_{USAN}}{\pi R^2} = \frac{1}{2} - \frac{|a|\left(d_2{}^3 - d_1{}^3\right)}{3\pi R^2} - \frac{\alpha}{\pi} + \frac{\sin(2\alpha)}{2\pi} \qquad (6)$$

Other case, where $a>0$ (which corresponds to a cavity point), the salient degree S_d is equal to:

$$S_d = \frac{A_{USAN}}{\pi R^2} = \frac{1}{2} + \frac{a(d_2{}^3 - d_1{}^3)}{3\pi R^2} + \frac{\alpha}{\pi} - \frac{\sin(2\alpha)}{2\pi} \qquad (7)$$

For $a=0$, S_d is equal to:

$$S_d = \frac{A_{USAN}}{\pi R^2} = \frac{1}{2} \qquad (8)$$

Finally, the relationship between S_d and a can be represented on a graph (see Fig. 2.). This relationship $S_d = h(a)$ is bijective.

Conclusion, for a fix value R and in the continuous domain, the salient degree S_d depends only on the value of the curvature a.

2.3. Study of the discontinuous case

The saliency degree is defined for a sufficiently smoothed surface (continuous case). If the number of points describing the curve decreases, the curve can no longer be considered as continuous but as a discontinuous one. The curvature can not be established on points of discontinuity (infinity). In the circular window, the saliency degree computation depends only on the edge curve of the object. It can be established at a discontinuous point. The discontinuous edge curve of an object can be modeled by:

$$f(x) = b|x| \qquad (9)$$

As presented in Figure 3 for $b<0$, the USAN area corresponds to the surface of the intersection between the circular window and the curve (in gray in the figure).

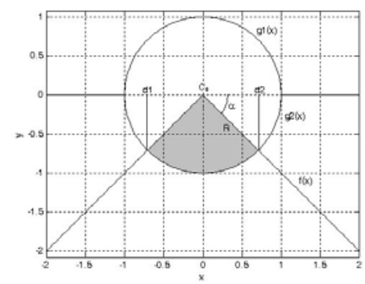

Fig. 3. Intersection of a circular window and a not sufficiently smooth surface with $f(x)=-|x|$ and R=1.

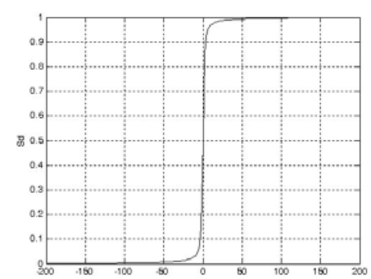

Fig. 4. Evolution of the saliency degree as a function of the parameter *b*.

We define d_1 and d_2 as the x coordinates of the intersection between the circular window and the *f(x)* function:

$$d_1 = -\sqrt{\frac{R^2}{(b^2+1)}} = -d_2 \qquad (10)$$

α corresponds to the angle between the x axis and *f(x)* (see Fig. 3.) and it is defined by:

$$\alpha = \arccos\left(\frac{d_2}{R}\right) = \arctan(b) \qquad (11)$$

Then, the Saliency degree is equal to:

$$S_d = \frac{A_{USAN}}{\pi R^2} = \frac{1}{2} - \frac{\alpha}{\pi} \qquad (12)$$

Other case, where $b>0$, (which corresponds to a cavity point), the salient degree S_d is equal to:

$$S_d = \frac{A_{USAN}}{\pi R^2} = \frac{1}{2} + \frac{\alpha}{\pi} \qquad (13)$$

For $b=0$, S_d is equal to:

$$S_d = \frac{A_{USAN}}{\pi R^2} = \frac{1}{2} \qquad (14)$$

Finally, the relationship between S_d and b can be represented on a graph (see Fig. 4.). This relationship $S_d = h(b)$ is bijective.

Conclusion, for a fix value R and in the discontinuous domain, the salient degree S_d depends only on the value of the parameter *b*.

2.4. Conclusion on S_d

The salient degree S_d is defined in the continuous and the discontinuous domain. The 3 types of edges (salient, flat and cavity) can be identified by the bijective response of the operator. Then, the value of S_d can be used to detect if the point C_0 is on a salient point or not.

3. SUSAN OPERATOR EXTENSION FOR MESHES

3.1. Introduction

In the previous chapter, the evolution of the saliency degree can be observed for a circular window centered at the C_0. Indeed, we are interested to apply this principle to detect salient points on a 3D mesh. A mesh always describes the edge of an object or a part of an object. At a point of the mesh of an object on a sufficiently smooth surface, the SFM techniques extract the curvature by integration, as described by D.L. Page [8] and expressed by Desbrun et al. [2] as "for every unit direction e_θ in the tangent plane, the normal curvature $\kappa^N(\theta)=\kappa_1\cos^2(\theta)+\kappa_2\sin^2(\theta)$, defined as the curvature of the curve that belongs to both the surface itself and a perpendicular plane containing both the normal vector and e_θ." The mean curvature $\overline{\kappa}$ is then defined as the average of the normal curvatures:

$$\overline{\kappa} = \int_0^{2\pi} \kappa^N(\theta)d\theta \qquad (15)$$

As proven in section 2.2., the saliency degree is related to the curvature for sufficiently smoothed surfaces. By analogy, figure 1 and 3 can be viewed as a visualization of a tangent plane at the point C_0. The saliency degree can be computed in every directions e_θ. Then, we define for a 3D model a mean saliency degree. At a point of a mesh S_d can be expressed by:

$$\overline{S_d} = \int_0^{2\pi} S_d^N(\theta)d\theta \qquad (16)$$

Where the normal saliency degree could be defined as $S_d^N(\theta)=S_d^1\cos^2(\theta)+ S_d^2\sin^2(\theta)$, with S_d^1 and S_d^2 the principal saliency degrees of the surface at C_0 and e_1 and e_2 the orthogonal principal directions of saliency.

The computation of the mean curvature is not possible in the presence of a discontinuity, but the mean saliency degree will always give a coherent result.

3.2. Intersection computation

The mean saliency degree computation, because of the isotropy of the circular analysis window in the e_θ directions, depends in fact on the volume intersection of a sphere and the surface around the inspected point. The 3D operator homologous to a circular pixellized window is a voxellized sphere. At a point of a mesh, the mean saliency degree is computed by determining the USAN volume. The nucleus of the voxellized sphere is defined as the center of gravity of the central voxel weighted by its volume. All voxels are defined as a point and a weight. By centering the nucleus at a point of the mesh, and defining the attribute of voxels under or on the surface as 1, the USAN volume can be determined by summing the weight of the voxels of the sphere that are under the surface. The saliency degree is finally defined by:

$$\overline{S_d} = \frac{V_{USAN}}{V_{Sph.Vox}} \qquad (17)$$

3.3. Functional scheme and prerequisite

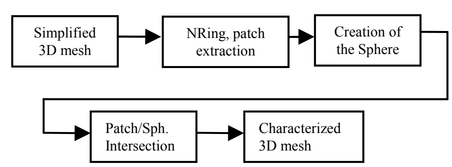

Fig. 5. Functional scheme of the3D SUSAN operator.

The mesh can be regular or irregular. The normal of each triangular face must be known, and compulsory oriented to the exterior of the object. To simplify the intersection computation, a patch around each point is extracted by an NRing method. The only radius for the sphere is globally determined for the object under inspection. To ensure a closed intersection between all patches and the sphere, the radius is adapted to the patch of smallest faces present in the mesh:

$$R_{Sph.Obj.} = \min_{i=0,M}\left[\min_{k=0,P}\left[\sqrt{\left(PtNRing_{-1}(k) - PtMes(i)\right)^2}\right]\right] \qquad (18)$$

With *N* equal to the number of Ring around each point, *M* the number of points of the object and *P* the number of points of N^{th} $Ring_{-1}$.

4. RESULTS

Figures 6 and 7 show saliency results obtained on three different objects. Salient points appear in red, cavity points in blue and flat points in green. Whatever the resolution (according to the number of points) of the mesh, the operator detects the characteristics of points of the objects: the icosahedron has a regular mesh, the flint artifact is a scanned and simplified object, and the Stanford dragon has an irregular mesh with a large number of points.

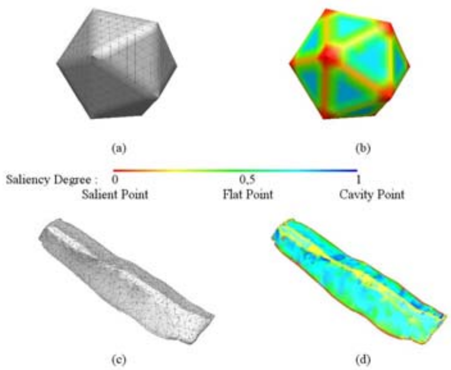

Fig. 6. (a) and (c) respectively the icosahedron and the flint artefact and their extracted characteristics (b) and (d).

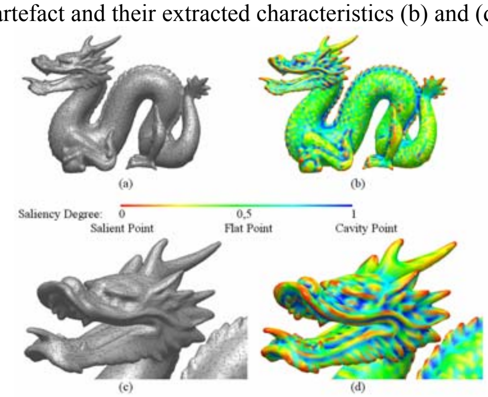

Fig. 7. (a) and (c) Stanford dragon and its extracted characteristics (b) and (d).

5. CONCLUSION AND PERSPECTIVES

In this article, a new measure is introduced, the saliency degree, which is related to the curvature as proved in section 2. This salient degree is defined for discontinuous forms for which the classical curvature measurement is not possible. The extension of this measure to 3D meshes has been presented. The point characterization is obtained without curvature computation. The only parameter entered by the user can simply be determined by a brief outline of the object before applying the algorithm. The saliency degree is reliable and sufficient to classify the 3 types of edges, salient, flat and cavity. However, the principal saliency degrees, representative of the principal curvatures, and the principal saliency directions, representative of the principal curvature directions, are not extracted by the operator. The comparison of our technique with others is already in progress. Future works are naturally focused on the extraction of the principal saliency degrees and directions. The influence of the noise on the characterization and a multi-scale scheme will be also studied.

6. REFERENCES

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